VORTEX EVOLUTION BY STRAINING:

A MECHANISM FOR DOMINANCE OF STRONG INTERIOR ANTICYCLONES

M. Montgomery – Naval Post Graduate School

J. McWilliams – UCLA
L. Graves – UCLA
Jupiter’s Red Spot
[IO over red spot and Europa shown to the right]
3 Earths could fit in red spot – at least 400 years old!
COHERENT VORTICES

1. Exist in nature:
   – Hurricanes
   – Tornadoes
   – Gulf Stream Rings
   – Red Spot [most famous (?)]

2. Also emerge in flows away from boundaries. These too are strongly influenced by planet’s rotation & stable stratification. But harder to see visually.

3. Observed in numerical studies
   – Random initial vorticity spectrum
   – Repeated mergers creating smaller number of larger vortices
   – Persistence (robust to disturbances – tend to axisymmetrize)
3-D BE Turbulence

Relative Vorticity

- Rotating, stratified turbulence
- Freely evolving 3D Gradient Wind Balance Equation model
- IC: Random isotropic wavenumber spectrum
- Periodic boundary conditions

Yavneh et al '97
Example solution from:

• Rotating, stratified turbulence
• Freely evolving 3D Gradient Wind Balance Equation Model
• IC: Random isotropic wave-number spectrum
• Periodic BCs

SHOWS:

• Vortex coherence / emergence
• Weakening (though perhaps hard to see here)
• Anticyclonic dominance (faster merger / alignment, less weakening)

• Motivates our current path – chasing anticyclonic dominance for interior flow regimes
1. McWilliams et al ’94 showed random vorticity field turbulently evolved through merger/alignment to one large cyclonic vortex and anti-cyclonic.

2. But QG not parity different! Answer lies at finite Ro.

3. [3D periodic cube, beta plane, Ro expansion] (Approx of PE that filters fast wave-speeds – gravity, inertial waves - since these motions exhibit weak dynamic coupling with slow motions that dominate).

4. Merger/alignment of vorticity into single vortex.

5. Anticyclonic merger/alignment faster and with larger vorticity

(see skewness for quantitative measure)
3DBE – Vortex mergers and alignment
Vortex Evolution By Straining And Anticyclonic Dominance

QUESTIONS

1. Do coherent vortices in nature weaken with time once they are formed? Why? How?

2. Apart from vertical boundary effects and the constraint of centrifugal stability (one-signed potential vorticity), why are anticyclonic vortices more common, and longer lived than cyclones?
VORTEX AXISYMMETRIZATION

- Axisymmetric azimuthal circulation is a stationary solution to the conservative fluid equations in gradient-wind and hydrostatic balance.

- Vortices are robust to perturbations that deform them from non-axisymmetric state.
  1. Melander, et al. 1987
Background to understanding vortices: Coherence & Persistence

1. Axisymmetric azimuthal circulation is a stationary solution to conservative fluid equations in Gradient Wind and Hydrostatic Balance

2. Vortices tend to be robust to small but finite amplitude perturbations that deform them from axisymmetric state, e.g., Melander et al., 1987 & Montgomery and Kallenbach, 1997

Example figure from Melander et al. shows vorticity that is initially elliptical by some deformation. Elliptical vortex then rotates and ‘sheds’ vorticity filaments that wrap around vortex
Axisymmetrization is perhaps the most important component in the development and maintenance of coherent vortices.

Typically a strengthening mechanism: Orr mechanism.
PREVIOUS PAPERS NOTING ANTICYCLONIC DOMINANCE  
(by freely decaying turbulence studies)

• Cushman-Roisin and Tang, 1990

• Aria and Yamagata, 1994

• Polvani et al., 1994

These free-decay turbulence studies with shallow water models observed anticyclonic preferential emergence

No comprehensive explanation however:

1. Either no attempt to explain
   OR
2. Only explain specific examples
FIG. 5. Plots corresponding to run 1 of Fig. 4. The initial parameters are \( L = 2L_0, \delta H = 0.4H, \) and thus \( E = 0.10 \) and the grid is \( 64 \times 64 \). The upper panels compare the \( \eta \) fields at the initial and final times \( (t = 0 \) and \( t = 3750 \) or 94 intrinsic time scales). The lower panel displays the initial and final spectra. Note the emergence of a few large eddies, all being anticyclonic, the decrease in length scale in the turbulent background, and the peak-and-tail final spectrum.

**Anticyclone** — solid

**Cyclone** — dash

Cushman-Roisin & Tang 90
Cushman-Roisin and Tang, 1990 Figure

• Just an example to illustrate previous anticyclonic dominance work

• One of the first to note anticyclonic preferential emergence

• Set up problem with many small vortices (equal # of cyclonic and anticyclonic) [seen here by height anomalies]

• Initially total energy peaks with scale with $L > L_d$
Briefly Strained Barotropic Vortex

Mean Enstrophy

Mean Energy

Time After Strain Period
**Bassom and Gilbert 1999, JFM**

- Axisymmetric vortex under external strain in BT model

- Strain Period
  1) Strain induced perturbations from mean vortex
  2) Weakens mean vortex

- Recovery Period (Strain Off)
  1) Interaction between waves and mean vortex
  2) Strengthens mean vortex

- Net change is vortex weakening
  (Seen in plots of *mean* energy and enstrophy with negative values)

- BT Model only!
  No finite Ro effects => No anticyclonic dominance

- But BG99 is an important paradigm:
  After studying vortex dynamics extensively, we have come to understand that this process is the controlling one for vortex weakening and anticyclonic dominance in geophysically relevant regimes characterized by finite Ro, and \( L_D \leq L \)
Model Formulation

Seek simplest model relevant to the questions posed.

- Rotating, conservative, Shallow Water Equations (SWE)

- Asymmetric and Gradient-Wind Balance Approximations

- Quasi-Linear Theory (linearized waves, nonlinear mean momentum and mass fluxes)

- Impulsive external strain (until $t = t_{max} = 0.1 \frac{L}{V}$)

- Rossby number $= \frac{V}{fL} = 0.2$ (Cyclonic $\text{Min}(h) \ll H$)

- (Deformation Radius)/(Vortex Size) $= \frac{L_d}{L} = 1.1$
Coherent Vortex In A Transient Strain Flow
Cartoon of Circular Vortex in Strain Field

• Strain field is non-divergent and irrotational (no vorticity)

• Strain is impulsive in time, smoothly ramping on and off over time interval small compared to free evolution of VRWs (latter occurs on differential advective time scale L/V)

• \( \delta \) simulates delta-function or impulse

• Is representative of far-field effects from spacially remote flows (vortices) after subtracting bulk motion of vortex as a whole
PERTURBATION GEOPOTENTIAL TENDENCY EQUATION

\[ D[\nabla^2 \phi' - \gamma^2 \phi'] - \frac{1}{r} \partial_r \phi' = -\frac{\eta^2}{r} \partial_r \left( \frac{r}{\eta} (\bar{\nabla} u_e' + \bar{\Omega} \partial_\lambda v_e') \right) + \frac{\Omega}{r\xi} (2\partial_\lambda v_e' - \partial^2_\lambda u_e') - \delta_e' \]
\[ + \gamma^2 \left[ \frac{\partial_r \phi'}{\eta} (f u_e' - \bar{\Omega} \partial_\lambda v_e') + \bar{\Omega} \partial_\lambda \phi_e' \right] \]

<= Strain Forcing

VORTEX-CHANGE GEOPOTENTIAL TENDENCY EQUATION

\[ \frac{\gamma^2}{r} \partial_r \left( \frac{r}{\gamma^2} \partial_\lambda \partial_\lambda \phi \right) = \gamma^2 \partial_\lambda \phi \]
\[ = \gamma^2 (\Sigma N^\sigma) - \frac{\gamma^2}{q} \partial_r \left[ \frac{r}{q} \langle \Sigma N^r \rangle \right] - \Gamma [\partial_\lambda \langle \Sigma N^r \rangle] \]

where

\[ \Gamma[Q] = \Gamma_{BE}[Q] \equiv \frac{\gamma^2}{r} \partial_r \left[ \frac{r}{q} Q \right], \]

\[ \Sigma N^u = \left\{ \begin{array}{l} u'_i \partial_r u'_i + \frac{1}{r} v'_i \partial_r u'_i - \frac{1}{r} (v'_i)^2 + \\
\frac{1}{r} v'_i \partial_r u'_i + \frac{1}{r} v'_i \partial_\lambda u'_i - \frac{1}{r} v'_i v'_i + \end{array} \right\}, \]

\[ \Sigma N^v = \left\{ \begin{array}{l} u'_i \partial_r v'_i + \frac{1}{r} v'_i \partial_r v'_i + \frac{1}{r} u'_i v'_i + \\
\frac{1}{r} v'_i \partial_r v'_i + \frac{1}{r} v'_i \partial_\lambda v'_i + \frac{1}{r} u'_i v'_i + \end{array} \right\}, \]

\[ \Sigma N^\phi = \left\{ \begin{array}{l} \frac{1}{r} \phi'_i \partial_r (ru'_i) + u'_i \partial_r \phi'_i + \frac{1}{r} \partial_\lambda (v'_i \phi'_i) + \\
\frac{1}{r} \phi'_i \partial_r (ru'_i) + u'_i \partial_r \phi'_i + \frac{1}{r} \partial_\lambda (v'_i \phi'_i) + \end{array} \right\}, \]

and

\[ \Sigma N^o = \left\{ \begin{array}{l} \frac{1}{r} \phi'_i \partial_r (ru'_i) + u'_i \partial_r \phi'_i + \frac{1}{r} \partial_\lambda (v'_i \phi'_i) + \\
\frac{1}{r} \phi'_i \partial_r (ru'_i) + u'_i \partial_r \phi'_i + \frac{1}{r} \partial_\lambda (v'_i \phi'_i) + \end{array} \right\}. \]
\[ \tilde{\nabla}^2 = \frac{\nabla^2}{r} \partial_r \left( \frac{r}{\tilde{\nabla}^2} \partial_r \right) + \frac{\nabla^2}{r^2} \partial_i^2 \]
\[ D = \partial_t + \frac{\tilde{\eta}}{r} \partial_r \]
\[ \delta'_c = \frac{1}{r} \partial_r (ru'_c) + \frac{1}{r} \partial_\lambda u'_c \]
\[ \mathcal{G}(r) = \frac{\tilde{\eta}}{\tilde{\xi}} \partial_r [\ln q] \]
\[ \tilde{\eta}^2 = \frac{\tilde{\eta} \tilde{\xi}}{\tilde{\eta} \tilde{h}} \]
\[ \tilde{\xi} = \frac{\tilde{\eta} \tilde{h}}{\tilde{\eta} \tilde{h}} \]
\[ \tilde{\zeta} = \frac{1}{r} \partial_r (ru'_i) \]
\[ \tilde{\Omega} = \frac{\tilde{v}_i}{r} \]
\[ \tilde{\kappa} = \frac{H_0}{\tilde{\kappa}} + \frac{1}{\tilde{\delta}_c} \]
\[ \tilde{\varphi}^2 = \tilde{\varphi}^2_{A,B,c} = 1 \]

and \( \langle \cdot \rangle \) represents an azimuthal average.

For times \( t \) satisfying \( 0 \leq t \leq t_{\text{max}} \), the external velocities are set by
\[ u'_c = -\alpha \tau \cos(2\lambda) \sin^2(\frac{t}{t_{\text{max}}}) \]
\[ v'_c = \alpha \tau \sin(2\lambda) \sin^2(\frac{t}{t_{\text{max}}}) \]
and \( \phi'_c \) satisfying gradient wind balance.

The norms are defined by
\[ E_{uw}[u, v, \phi] = \frac{1}{2} \left( h(u^2 + v^2) + \frac{1}{g} \phi'^2 \right) \]
\[ E_{uw}[u, v, \phi] = \frac{1}{2} h q^2 \]
\[ E_{uw}[u, v, \phi] = \frac{1}{2} h q^2 \]
Initialize model using continuous, monotonic relative vorticity profile $\bar{\zeta}(r)$

- Parameters chosen so $V_{\text{max}} = 1$ at $r = 1$

- All experiments initialized with identical profile of relative vorticity

1. Change sign of $f$ as desired for cyclones or anticyclones
2. Change magnitude of $f$ to change $Ro$
3. Change magnitude of $f$ and $H$ to change $L_D$
Fluctuation Response To Impulsive Strain

\[ \phi' \quad (t = t_{\text{max}}) \]

\[ r/L \]

solid - CYCLONE

dashed - ANTICYCLONE
dash dot - OS

\[ \phi = \phi'(r) M(t) fVL \cos 2\lambda \]

\[ M(t) = \frac{2m}{t_{\text{max}}} \int_0^t F(t') dt' \]

\[ m = a t_{\text{max}}/2 \ll 1 \]
Fluctuation Response to Impulsive Strain

Shown is radial component of wave #2 perturbation caused by interaction of external strain on mean vortex

Larger response in cyclone vs anticyclone (QG in between)

Larger cyclone response due to larger value of $\bar{\eta}$ (abs. vorticity $[f + \zeta]$, $\bar{\xi}$ (twice abs. rotation rate $[f + 2\Omega]$) and fact that forcing terms are additive for cyclone, competitive for anticyclone (cf. force balance, this effect is amplified!)
Vortex Change Response To Impulsive Strain

\[ V \]

solid = CYCLONE

dashed = ANTICYCLONE
dash dot = QG

\[ \frac{1}{V_m^2} \]

\[ \phi \]

\[ \frac{1}{V_f L m^2} \]
Vortex Change Response to Impulsive Strain

• Due to interaction of perturbations with mean and strain
  $-\langle V \rangle$ -- all results are negative --- weakening
  $-\langle \phi \rangle$-- all results are positive --- weakening

  (anticyclone here multiplied by negative sign, so,
  like cyclone in which positive value reduces mean
  low (negative) )

• External strain weakens vortices

• Cyclone has greater weakening than anticyclone (QG in
  between), forcing terms additive in cyclone, competitive in
  anticyclone
VRW Relaxation Phase (axisymmetrization)
Spiral Plot of Wave PV for Cyclone

• Strain turned off

• Strain induced wave #2 perturbation (first panel)

• Vortex relaxation begins on differential advective time scale
  ≈ \( L/V \)

• Differential mean angular velocity (\( \Omega \)) causes azimuthal shear
  and waves (VRWs) wind up spirally

• Filamentation (due to spiral windup)

• Axisymmetrization
\[ \bar{G}(r) = \bar{\xi}(r)d \ln \bar{q}(r) / dr \]

Figure 8: \( \bar{G}(r) \) normalized by \( f/L \) in CYCLONE (solid line) and ANTICYCLONE (dashed line).
Figure 9: Decay of $|\phi'|$ at the radius of maximum mean potential vorticity gradient ($r \approx 0.6L$) during the vortex relaxation phase (starting at $t = t_{max}$), normalized by its value at $t_{max}$. CYCLONE = solid line; ANTICYCLONE = long-dash line; QG = dash-dot line; BT = dotted line.
Vortex Change Response During Relaxation Phase

\[ \langle V \rangle \]

- solid = CYCLONE
- dashed = ANTICYCLONE
- dash dot = QG

\[ \frac{r/L}{m^2 V} \]

\[ \frac{\phi}{m^2 V/L} \]

\text{~ Pseudo-Momentum RULE}
Vortex-Change Response During Relaxation Phase

- Waves interact with mean vortex during axisymmetrization process
- Azimuthal shear tilts phase lines in direction of tangential wind ($v$)
- Divergence of azimuthally averaged eddy momentum flux (Reynolds’ stress)
  - becomes negative on coreward side of wave
  - becomes positive on outward side
- Change in azimuthal wind ($v$) negatively proportional to eddy momentum flux
  \[ \frac{dv}{dt} \approx -\nabla \cdot (w v') \]
- Positive peak in $\langle V \rangle$ in core negative outside
- Larger peak in core due to conservation of angular momentum.
- Core undergoes a net strengthening process
- Larger strengthening for cyclone!
  1. Strain induced pert. larger in cyc, therefore more relaxation
  2. Larger divergence of eddy momentum flux (averaged)
Net Vortex Change

• Combination of strain weakening and strengthening

• Net signal is weakening in core for all cases

• Cyclone has more weakening than anticyclone

NOTE:
<V> shows anticyclone has greater peak but <ϕ> values more important for energy/enstrophy norms when L/L_D ~ O(1)
Vortex Change

\[ \Delta \text{MEAN ENERGY NORM} \]

\[ E = \frac{1}{2} \int (u^2 + v^2) + \frac{2}{3} H \]

\[ t \frac{V}{L} \]

\[ \Delta \text{MEAN ENSTROPHY NORM} \]

\[ E_{\text{enstrophy}} = \frac{1}{2} \int h \varphi^2 \text{d}A \]

\[ t \frac{V}{L} \]
Mean Vortex Change Norms

• $t = 0$ refers to end of strain period
All values plotted after end of strain period.

All values **NEGATIVE** $\rightarrow$ net vortex weakening for all cases

• Relaxation phase (strengthening) becomes mild after approx. one eddy turnaround time ($=2\pi$), i.e., fluctuations approach spiral wind up regime and close to being aligned with shear – thus no further energy transfer to mean

• Larger cyclone weakening than anticyclone (QG in between)

• Anticyclones show greater robustness to strain events

• We have extensively explored the parameter space for vortex in straining events and results shown are typical and answer the question posed

\[
E = \frac{1}{2} \int \left[ h(u^2 + v^2) + \frac{1}{g} \phi^2 \right] dA \\
E\text{,w} = \frac{1}{2} \int h\phi^2 dA
\]
What about **SUSTAINED STRAIN**?

- Same IC as before

- Same onset of external strain, then held fixed at the maximum value thereafter
Late Time Wave PV In Sustained Strain

CYCLONIC $q^{00}$ (contour interval 0.117)

ANTICYCLONIC $q^{00}$ (contour interval 0.025)

[AC, sign reversed]
Late Time Wave PV Spiral Plots for c/ac vortex in sustained strain

• Mixture of straining and relaxation phases

• VRWs attempt spiral windup, but impeded by fixed strain

• No axisymmetrization; waves locked in 45° orientation to strain (standing waves) where there is balance between VRW wrap-up and external strain

Details:
– Some wrap-up up seen: fine structure in core
– Note existence of perturbation PV in outer region of opposite sign for anticyclone

Overlay strain cartoon to see strain-induced eddy PV fluxes:
– Positive PV part inward (core strengthening)
– Negative PV part outward (core strengthening)

→ Mechanism responsible for anticyclonic growth in this model
Wave PV In Sustained Strain

CYCLONIC max \[\Phi^C\]

ANTICYCLONIC max \[\Phi^A\]

\[r/L\]

\[\Phi/L\]
Hovmuller Plots of Wave PV for Cyclone/Anticyclone in Sustained Strain (normalized by max value)

• VRWs now interact with external strain as well as with vortex

• Radial propagation of outer PV pert – no stagnation radius, more propagation for anticyclone

• Observe similar $\phi'$ as in strain impulse case

• Opposite signed PV pert at larger radius yields sustained growth for ac vortex!
Vortex Change In Sustained Strain

\[ \langle V \rangle \]

\[ r/L \]

solid - CYCLONE

dashed - ANTICYCLONE
Vortex Change In Sustained Strain

• Net weakening for cyclone

• Net strengthening for anticyclone

• Occurs in conjunction with outer PV perturbation
Vortex Change In Sustained Strain

**Mean Energy Norm**

**Mean Enstrophy Norm**

- **Solid** - Cyclone
- **Dashed** - Anticyclone
- **Dash Dot** - OG
Mean Vortex Change Norms for Sustained Strain

• General behavior (shown by vortex core) is continual weakening

• Finite Ro effects
  – larger weakening for cyclones
  – strengthening for anticyclones

• Longer strain events lead to greater and greater cyclone/anticyclone difference
  => greater anticyclonic dominance
Fully Nonlinear Tests of Quasilinear Predictions

Use Nonlinear Balance Equations in 3D
(e.g., Yavneh et al. ’97)
ac elliptical vortex
ac elliptical vortex undergoes axisymmetrization
c elliptical vortex
undergoes axisymmetrization
ac vortex merger
ac vortex merger
cyclonic vortex merger
SUMMARY

• Strain events lead to net vortex weakening even though VRW relaxation and axisymmetrization is strengthening

• Anticyclones are more robust – cyclones are more susceptible to weakening by external strain, leading to anticyclonic dominance over multiple straining events

• Under sustained strain, cyclones continue to weaken, but anticyclones strengthen. While a sustained strain may be unphysical, this result hints that under somewhat prolonged straining events (say by other vortices) anticyclones may actually grow