Overflows and Convectively-driven flows

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Lecture 2: Overflow processes: focus on entrainment
Summary of overflow processes

Final properties, transport and depth of overflow product water depend on all these processes.

- Shear instability
- Neutral buoyancy level
- Geostrophic eddies
- Entrainment of ambient water
- Bottom friction
- Downslope descent
- Hydraulic control

What are the causes and consequences of entrainment?
Evidence for entrainment

- Increase in transport downstream
- Dilution of salinity/temperature/density anomaly of overflow downstream

Faroe Bank Channel overflow

Transport in different T,S classes

Broad slope section

Sill section
Entraining stream tube model of overflows

\[
\frac{d}{dx} \int_U h dy = \int w_e dy
\]
Conservation of mass

\[
\frac{d}{dx} \int_U h q dy = \int w_e q_e dy
\]
Tracer conservation

\[
w_e = EU
\]
Entrainment velocity
Diagnosing entrainment rate

\[
\frac{d}{dx} \int_L Uhdy = \int_L w_e dy \quad \rightarrow \quad E = \frac{w_e}{U} = \frac{d}{dx} \int_L hUdy
\]

Transport method

\[
\frac{d}{dx} \int_L Uhqdy = \int_L w_e q_e dy \quad \rightarrow \quad E = \frac{w_e}{U} = \frac{h}{(q_e - q)} \int_L dq
\]

Tracer method
Estimating entrainment rate from numerical simulations

Simulation of idealized Denmark Straits-like overflow

In this scenario, most entrainment occurs in first 150km after flow exits channel.

\[ E = \frac{w_e}{U} = \frac{\frac{d}{dx} \int hUdy}{UL} \]
Entrainment mechanisms

2D nonrotating simulation: Ozgokmen and Chassignet, 2002

For oceanic overflows in quasi-steady state, the Kelvin-Helmholtz billows in the extended tail are more relevant.
Entrainment mechanisms

3D nonrotating gravity current simulation. Ozgokmen et al, 2004

Salinity
Entrainment mechanisms

Snapshots of mixing in downslope current

Cross-flow Vorticity

Rollups of dense fluid over light

Lab experiments, Pawlak and Armi, 2000
Entrainment mechanisms

How can buoyancy-driven flow lead to shear instability?

From linear stability theory, instability is possible if $\text{Ri} = \frac{N^2}{(dU/dz)^2} < 1/4$
Energetics of downslope dense current

For a 2D, nonrotating dense layer in steady state, on a slope, with no entrainment or friction

\[
\frac{d}{dx}(U^2 h) = -h g' \sin \theta
\]

\[
\frac{d}{dx}(U h) = 0
\]

\[
\frac{d}{dx} g' = 0
\]

g' = -g \left( \rho_B - \rho_A \right) / \rho_0 < 0

Kinetic energy gain after vertical distance H:

\[
\Delta KE = -\rho_0 g' H = -\Delta PE
\]

With mixing and friction some KE will be lost to mixing (increasing PE) and dissipation
Flow chart for energy in a dense overflow with mixing and friction

Large-scale available potential energy

\[ \rightarrow \]

Large-scale kinetic energy of dense current

\[ \rightarrow \]

Dissipation to bottom friction

\[ \rightarrow \]

Instability

\[ \rightarrow \]

Dissipation through small-scale motion

\[ \rightarrow \]

Mixing increases potential energy
If flow is in geostrophic balance, then $$U = \frac{g'}{f} \tan \theta$$

Geostrophic dense currents do not descend slope and accelerate indefinitely, but rather move along isobaths.
Cross-isobath transport in rotating overflows

Friction breaks geostrophic balance, allowing downslope flow.
Laminar flow, no mixing, when

\[ Ek = \left( \frac{\delta_E}{h} \right)^2 \geq 0.1 \quad \delta_E = \sqrt{\frac{2\nu}{f}} \]

\[ Fr = \frac{U}{\sqrt{g'h}} < 1 \]

Growing lateral instability allows downslope (and upslope) flow.

Cenedese et al, 2004

\[ Ek < 0.1 \]
Roll-wave regime

For large $Ek$ and moderate to large $Fr$, roll-wave develop, which break and cause mixing.

Cenedese and Adduce, 2008

Cenedese et al, 2004
Flow transitions through different regimes

For large Fr, flow is turbulent

Cenedese et al, 2004
Regime diagram for rotating dense currents

\[ E_k = \left( \frac{\delta_E}{h} \right)^2 \]

\[ Fr = \frac{U}{\sqrt{g' h}} \]

Cenedese et al, 2004
Mixing in hydraulic jumps

Temperature

Subcritical flow: $Fr = \frac{U}{\sqrt{g' h}} < 1$

Supercritical flow: $Fr = \frac{U}{\sqrt{g' h}} > 1$

Transition at sill: hydraulic control

Transition to subcritical flow: hydraulic jump

Kelvin-Helmholtz billows in transition region
Transverse hydraulic jumps in presence of rotation

Transition from supercritical to subcritical state leads to sudden widening of current, also associated with change in mixing regime.

Pratt et al, 2007, numerical simulations of Faroe Bank Channel overflow

Riemenschneider and Legg, 2007
Enhancement of entrainment by rough topography

Salinity and velocity from 3D nonrotating simulations (Ozgokmen et al, 2008)

When roughness height $\rightarrow$ dense layer thickness, then entrainment is enhanced.
Summary of entrainment mechanisms

• Shear instability and resultant turbulence is primary cause of entrainment.
• Shear is generated by the acceleration of dense flow down slope.
• In rotating flows, entrainment can occur through roll-waves and laterally, through mesoscale eddies, as well as shear instability, depending on parameter regime.
• Localized entrainment occurs in hydraulic jumps.
• Rough topography enhances entrainment.
In a bulk view of the overflow plume, entrainment is parameterized in terms of bulk properties such as Froude number, e.g.

\[ E = \frac{0.08Fr^2 - 0.1}{Fr^2 + 5} \quad \text{for} \quad Fr^2 \geq 1.25 \]  

(Ellison and Turner, 1959)
Entrainment depends on Reynolds number too

Cenedese and Adduce, 2008
A new parameterization incorporating both low Fr entrainment, and Re dependence

\[ E = \frac{E_{\text{min}} + A F_r^\alpha}{1 + A C_{\text{inf}} (F_r + F_{r0})^\alpha} \]

\[ C_{\text{inf}} = \frac{1}{E_{\text{max}}} + \left( \frac{B}{\text{Re}^\beta} \right) \]

Cenedese and Adduce, 2010
Parameterizing entrainment in terms of gradient Richardson number

An entrainment parameterization in HYCOM is calibrated by comparing the entrainment rate with nonhydrostatic simulations (shown at left). The final calibrated entrainment parameterization can be expressed as

$$E = E_0 \left(1 - \frac{Ri}{Ri_c}\right)$$

with $E_0 = 0.2$ and $Ri_c = 0.25$ (Xu et al, 2006)
Parameterizing Entrainment

New parameterization of shear-driven mixing

\[
\frac{\partial^2 \kappa}{\partial z^2} - \frac{\kappa}{L_B^2} = -2SF(Ri) \quad F(Ri) = \frac{0.15(1 - Ri/Ri_c)}{(1 - 0.9Ri/Ri_c)}
\]

where S is the vertical shear of the resolved horizontal velocity, and \(L_B = Q^{1/2}/N\) is the buoyancy length scale (the scale of the overturns), N is the buoyancy frequency, and Q is the turbulent kinetic energy, found from an energy budget.

New parameterization contains no dimensional constants. Tuned by comparison with lab expts & high res. numerical simulations.

Could be modified to include low Fr (high Ri) mixing and Re dependence

3D high res simulation
MITgcm

Diffusivities diagnosed from simulation and predicted by different parameterizations (new parameterization = p1)

Jackson, Hallberg, and Legg (JPO, 2008).
Other influences on entrainment: small-scale topography

Plume thickness

Mehmet Ilicak
Summary of entrainment parameterization

• Bulk parameterizations of entrainment relate total entrainment to Froude number and Reynolds number, deriving empirical relationships.

• Local parameterizations relate mixing to gradient Richardson number/shear, similarly to 2-equation turbulence models.

• Still need improvement to represent low Fr mixing, mesoscale eddy processes, sub-grid topographic effects.
Detrainment

Salinity for different slope angles and stratification

Ozgokmen et al, 2006
Detrainment

**Intrusion depth**

Using entraining plume model, prediction for intrusion depth (where dense current has same density as surroundings) is found to be:

\[ Z \sim E^{-1/3} B^{1/3} / N \]

Where \( E \) = entrainment coefficient,
\( B \) = buoyancy flux in plume
\( N \) = ambient buoyancy frequency
A continuously detraining regime is observed when bottom drag counters increase in transport due to entrainment and gravity.

Baines, 2001
Parameterization of frictional bottom boundary mixing

With thick plumes both interfacial shear mixing and drag-induced near bottom mixing are needed. (Legg, Hallberg and Girton, 2006).

Observed profiles from Red Sea plume from RedSOX (Hartmut Peters)
Impact of frictional bottom boundary mixing on GCM results

Mediterranean outflow salinity: comparison between 1 degree Generalized Ocean Layer Dynamics (GOLD) Isopycnal Model and observed climatology.

New parameterization eliminates spurious bottom plume

Spurious bottom plume
Impact of cross-channel variations on mixing

Rotating dense flow in channel of width close to deformation radius, height close to Ekman thickness. Secondary circulation induced by rotation and friction influences entrainment.

Umlauf et al, 2010
Impact of cross-channel variations on mixing

Fluid entrained into interface is then transported by secondary circulation to opposite side of channel.

Umlauf et al, 2010
Summary: entrainment in overflows

- Many different processes in overflows affect entrainment, including: friction, shear instability, roll-waves, mesoscale eddies, topographic effects.
- Entrainment may be parameterized by a relationship to Froude and Reynolds number, but this may not capture all of the above regimes.
- Detrainment, fluid transport away from the boundary, is also important, and can be sensitive to the entrainment rate, as well as friction/drag.
- The plume structure is sensitive to the vertical extent of mixing, and a bulk model is not always a good representation.
- Lateral inhomogeneities (e.g. due to rotation, friction) can influence entrainment.