Buoyancy-forced circulation and downwelling in marginal seas

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Abstract

A review of buoyancy-forced circulations in marginal seas is presented. The focus is on the large-scale properties that characterize the water mass transformation and exchange, with additional discussion on the dynamics that support a net downwelling transport. The approach utilizes a combination of idealized nonlinear numerical model calculations and simple conceptual analytic models. Predictions of the primary measures of the exchange (temperature of convective water mass, temperature of outflowing waters, heat transport, net downwelling) provided by the analytic models compare well with nonlinear numerical model calculations across a wide range of parameter space. Eddy fluxes from cyclonic boundary currents into the basin interior play the central role in determining all characteristics of the exchange, including the mean strength of the cyclonic boundary current. The dynamics that support a net downwelling are identified for three distinct parameter regimes: dissipative stratified flows; weakly dissipative stratified flows; and weakly stratified flows. In all cases, the net downwelling transport is localized to within a deformation radius or less of the lateral boundary.
1. Introduction

Marginal seas subject to buoyancy loss, because of their semi-enclosed geometry, are source regions for the formation of dense intermediate and bottom waters. These convective water masses generally have distinct properties relative to the open ocean and can be traced far from their formation basins. They also can transport significant amounts of heat, salt, and other tracers throughout the world ocean. The vertical circulation and meridional heat and freshwater transports are fundamental components of the oceanic circulation, and play important roles in the global climate system. Understanding how this circulation depends on the environmental parameters of the system is important if one is to better model and predict the climate system and its sensitivity to changing atmospheric conditions, such as increasing anthropogenic carbon dioxide.

The focus of this review is on the circulation and exchange resulting from surface buoyancy-forcing in marginal seas. General characteristics of the exchange between the marginal sea and the open ocean are described from eddy resolving numerical models in idealized configurations and the physics governing this exchange are elucidated through a combination of numerical models and simplified analytic models. Although the problems are couched in terms of marginal sea - open ocean exchange, many of the processes that emerge from this analysis are relevant to more general buoyancy-forced flows. Particular attention is paid to the dynamics involved with net vertical motions forced by surface cooling. Distinct regimes of dissipative stratified flows, weakly dissipative stratified flows, and weakly stratified flows are covered. Analytical treatments of some aspects of these flows can be found in Chapter XX. Selected results from each regime are presented here, with the objective of identifying and distinguishing the key dynamics in each of these parameter regimes. The focus here is on the processes within the
marginal sea, mixing processes that lead to vertical transports and water mass modification as dense waters formed in marginal seas flow over topographic sills are not considered here (see Chapter XX).

2. Buoyancy-forced circulation and exchange

Previous studies of localized open ocean convection have been very useful for elucidation of the dynamics of convective plumes and the onset of a scale cascade from the plume scale to the mesoscale (see the review by Marshall and Schott, 1999, also Chapter XX). Because those studies were forced with buoyancy loss only, they could not achieve an equilibrium since there was no source of heat to balance the cooling. Observations also indicate that the majority of the exchange between marginal seas and the open ocean is carried out in narrow boundary currents that encircle the marginal sea cyclonically. These modeling studies also neglected wind-forcing, which may be an important driver of the observed cyclonic boundary current. In this section, we will build upon these previous localized convection studies to include: a source of heat (to achieve equilibrium solutions); boundaries to support boundary currents; topography; and wind-forcing.

The approach taken throughout this section will be to combine results from idealized configurations of primitive equation numerical models with less dynamically complete, but more dynamically revealing, conceptual and theoretical models. These simplified models are very useful because they will identify how key aspects of the circulation depend on specific environmental parameters. This generalization of the processes serves to unify our understanding of the dynamics of marginal seas, and provides simple interpretations as to why various marginal seas behave differently in terms of their exchange with the open ocean.
The numerical model used in the following studies is the MIT general circulation model (Marshall et al. 1997), which solves the primitive equations on a Cartesian, staggered C-grid with level vertical coordinates. Details of the model configurations vary, but there are some aspects in common. There is generally a marginal sea separated from an adjacent basin (referred to as the open ocean) by a sill or horizontal confinement. The marginal sea is subject to buoyancy loss and heat is provided in the open ocean to balance this cooling. All calculations discussed here use a linear relationship between temperature and density with a thermal expansion coefficient of \(0.2 \text{ kg m}^{-3}\text{C}^{-1}\). The models are initialized at rest with uniform stratification and run until a statistical equilibrium is achieved, typically 20-30 years.

Heat is provided in the open ocean by restoring the model temperature towards a uniform stratification with a time scale of \(O(20 \text{ days})\). This approach allows the model to achieve equilibrium on the dynamic time scale, which is typically of \(O(10 \text{ years})\) and is set by the time it takes eddies or Rossby waves to cross the basin. The equilibration time scale for basin-scale models that have buoyancy-forcing at the surface only is set by diapycnal diffusion, and is typically \(O(1000 \text{ years})\). However, the restoring approach used here does not allow the low latitudes to respond to changes in the marginal sea. The external restoring also provides potential energy to balance any loss of potential energy resulting from sinking of dense waters in the marginal sea. This is of course a gross oversimplification of what goes on to provide the stratification outside a marginal sea in the real ocean. This parameterization does, however, allow for a small computational domain, which allows for eddy resolving resolution, and removes the long time scale required to arrive at a thermodynamic equilibrium that is required when diapycnal mixing controls the stratification in the open ocean.
a. influence of a boundary

The effects of an open ocean adjacent to the marginal sea are demonstrated by a simple extension of the localized cooling experiments reviewed by Marshall and Schott (1999). The model domain consists of a circular marginal sea connected to a small open ocean through a narrow strait (a typical domain is shown in Fig. 1). In some cases, such as the one in Fig. 1, there is a linearly sloping bottom topography around the perimeter of the basin. The model is forced by a repeat annual cycle consisting of cooling for a period of 2 months in the interior of the marginal sea followed by 10 months with no surface forcing. There is also a continuous restoring of temperature towards a uniform stratification in the open ocean.

In a calculation similar to that of Spall (2003), the simplest extension of the localized open ocean convection configuration is provided by including lateral boundaries in a flat bottom ocean while keeping the heat loss localized in the interior of the basin. In this case, the stratification in the open ocean is restored to $N^2 = 4 \times 10^{-6} \text{s}^{-2}$, which, with the basin depth of 1000 m and uniform Coriolis parameter of $10^{-4} \text{s}^{-1}$, gives a baroclinic deformation radius of 20 km. The horizontal grid spacing is 5 km and there are 20 levels in the vertical with thickness 50 m.

A snapshot of the sea surface temperature 1 month after the end of the cooling period in year 24 is shown in Fig. 2. The heat loss in the marginal sea is limited to the region inside the white circle. Cooling results in deep convection, which drives a baroclinic current and initiates baroclinic instability, much as is found in the localized cooling experiments. However, because the open ocean is maintained to be warm, a density gradient develops between the interior of the marginal sea and the open ocean. This drives a warm inflow primarily along the boundary on the right hand side of the marginal sea (in the direction of Kelvin wave propagation). This boundary current becomes baroclinically unstable and sheds warm eddies into the basin interior. The density gradient between the interior and the boundary increases to the point where the
eddies shed from the boundary transport enough heat into the interior to balance the imposed surface cooling. At this point the model has reached a statistical equilibrium. This balance is different from the localized cooling experiments because the eddies are shed from the boundary current, not from the edge of the cooling region.

The inflow to the marginal sea takes place in the upper ocean with a compensating outflow of dense water in the deep ocean. This implies that downwelling is taking place somewhere within the marginal sea. An azimuthal integral of the downwelling within the marginal sea averaged over the final 10 years of the 25 year integration is shown in Fig. 3. The cooling region is indicated by the bold black line from the center of the basin out to 150 km radius. The downwelling is located adjacent to the outer boundary of the marginal sea, clearly separated from the cooling region. The net downwelling within the cooling region is essentially zero. The dynamics within the downwelling region will be discussed in Section 3.

This model configuration is not very realistic, but does represent an initial step in connecting the isolated convection studies to the basin- and global-scale circulations. The important points demonstrated by this calculation are that the boundary: 1) supports a warm boundary current that provides heat to balance cooling in the marginal sea and 2) permits a net vertical mass transport in the basin.

**b. influence of sloping topography**

Two aspects of this calculation that are not very realistic are the localization of surface cooling and the neglect of bottom topography. These limitations are relaxed in a model configuration that applies uniform surface cooling over the entire marginal sea and includes a sloping bottom around the perimeter of the basin (as in Fig. 1). This section follows the results of Spall (2004), see the original study for additional details. For purposes of illustration, a calculation similar to
the central calculation in Spall (2004) is reproduced here. The mean temperature and velocity at the uppermost level over the last 10 years of a 25 year integration are shown in Fig. 4. As for the case with no topography and localized cooling, the heat loss in the interior drives a warm, cyclonic boundary current. The interior is cool and nearly quiescent. The boundary current is confined to the region of sloping topography (indicated by the white line around the perimeter of the basin) and encircles the entire marginal sea. The outflowing water is cooler than the inflowing water, but not as cold as the convective water mass in the interior of the basin.

A vertical section of the mean meridional velocity and temperature at $y = 600$ km are shown in Fig. 5. The boundary current is marked by strong stratification and steeply sloping isopycnals over the bottom topography. The interior of the basin is much cooler throughout and is very weakly stratified. The maximum velocity in the boundary current is approximately 30 cm s$^{-1}$. However, unlike the case with a flat bottom, the velocity is now in the same direction throughout the water column, and is very weak at the bottom. The outflowing waters are cooler than the inflowing waters as a result of heat loss to the north of this section. The northward mass transport along the eastern boundary is balanced by the southward transport along the western boundary. This conservation of mass transport, combined with the cooler water on outflow, requires that the southward flowing water is more barotropic than the northward flowing water (the maximum velocity is slightly lower and the velocity extends slightly deeper in the water column), which in turn implies that there is a net downwelling north of this section.

The general characteristics of this velocity and hydrography section are consistent with what is observed in the upper 1500 m of the Labrador Sea. The potential temperature and absolute velocity along the AR7W section (approximately XXN) are shown in Fig. 6 (from Pickart and Spall, 2007).

The spinup of the circulation and hydrography are informative as to the dynamics of the
equilibration. Time series of temperature at the uppermost model level at the two locations indicated by the dots in Fig. 4 are shown in Fig. 7. The boundary current quickly develops a seasonal cycle in which warm water is flowing into the basin over most of the year with short spikes of cooler water at the beginning of the cooling period. The interior takes much longer to equilibrate. Early in the calculation, the interior temperature cools rapidly during the forcing period and remains at a nearly constant temperature when there is no surface forcing. This cycle repeats for several years, during which time the temperature of the interior waters continues to decrease. After about 5 years we see an increase in the interior temperature during the period when cooling is turned off, which implies an exchange with the open ocean via the boundary current. It takes about 15 years for the interior to achieve a statistical equilibrium, which is characterized by seasonal oscillations in temperature of $O(0.5 \, ^\circ\text{C})$.

The sea surface temperature and mixed layer depth at the end of the cooling period are shown in Fig. 8. The boundary current is cooler than the annual mean primarily as a result of direct heat loss to the atmosphere. The mixed layer is deep over most of the interior with the exception of a couple large patches of stratified water left over from before the cooling started. We can also see warm, stratified water beginning to penetrate into the interior from the boundaries in the form of small-scale meanders and eddies. One month after the end of the cooling period the warm water has penetrated much further into the basin and warm eddies are being shed from the boundary into the interior (Fig. 9). As a result, the mixed layer depth is less than 100 m over most of the interior.

This restratification process motivates the construction of a simpler conceptual model of the marginal sea that can be used to predict the main characteristics of the exchange between the marginal sea and the open ocean. The water masses in and around the marginal sea can be described by 3 water types: inflowing warm water, outflowing cool water, and the coldest...
water, which is found in the deep convection region in the interior of the basin. Several basic constraints can be reasonably imposed: mass conservation within the marginal sea; a closed heat budget within the marginal sea; and a closed heat budget within the interior of the marginal sea. It is assumed that the mean velocity field is in geostrophic balance with the mean density field. The final constraint derives from assuming that the heat loss in the interior of the marginal sea is balanced by lateral eddy heat fluxes that originate from the cyclonic boundary current.

This final constraint can be used to estimate the density of the water that is formed within the interior of the marginal sea, relative to that of the inflowing water (we get to chose a reference density that all other densities are relative to). Following Visbeck et al. (1996), a heat budget for the interior of the marginal sea can be written in terms of the mean surface heat loss $Q$ and the eddy heat flux $\overline{u'T'}$ as

$$PH_{in}\overline{u'T'} = \frac{AQ}{\rho_0 C_p}$$

where $P$ is the perimeter of the marginal sea, $H_{in}$ is the thickness of the inflowing water, $A$ is the surface area of the interior of the marginal sea, $\rho_0$ is a typical density of seawater, and $C_p$ is the specific heat of seawater. This balance implicitly assumes that the mean advection from the boundary into the interior is small. This is consistent with the model result, and with the notion that the mean flow tends to flow along geostrophic contours (bottom topography for this f-plane configuration).

The eddy flux is assumed to be proportional to the mean baroclinic velocity $V_{in}$ times the temperature anomaly of the boundary current relative to the interior as

$$\overline{u'T'} \propto V_{in}(T_{in} - T_0)$$

This functional relationship is supported by linear stability theory (Green, 1970; Stone, 1972).
and in a wide range of flat bottom model calculations and laboratory experiments, as summarized in Visbeck et al. (1996). Visbeck et al. (1996) find empirically that the constant of proportionality is approximately 0.025 for a wide range of applications.

In the present configuration the baroclinic boundary current is flowing over a sloping bottom, which we expect to alter the stability properties of the boundary current. The linear, quasigeostrophic theory of Blumsack and Gierasch (1972) provides a simple means to incorporate the effects of a sloping bottom into (2), as used by Spall (2004). The Blumsack and Gierasch (1972) solution for the linear growth rate of a uniformly sheared velocity field with uniform stratification over a sloping bottom is

\[ \eta_i = \left[ \frac{\mu - \tanh(\mu)}{\tanh(\mu)} (1 - \delta) - \frac{1}{4} \left[ \frac{\delta}{\tanh(\mu)} - \mu \right]^2 \right]^{1/2} \]  

where \( \mu \) is the nondimensional wavenumber and \( \delta \) is the ratio of the bottom slope to the isopycnal slope. The resulting growth rate is shown as a function of bottom slope and wavenumber in Fig. 10a.

The flat-bottom result is recovered for \( \delta = 0 \). The general effect of bottom topography is to stabilize the flow, although there is a small region for \( \delta \) slightly positive for which the growth rate increases relative to a flat bottom. For warm, cyclonic boundary currents, as found in convective basins, \( \delta < 0 \). The growth rate is strongly reduced by steeper topography and the wavelength of the most unstable waves shifts to smaller scales. The value of \( \delta \) for the model calculations is approximately -0.8 (Fig. 5), similar to that found for the Labrador Sea (Fig. 6).

Using the theory from Blumsack and Gierasch (1972), the eddy heat flux can be written as

\[ \overline{u'T'} \propto \frac{\tanh(\mu)}{\mu - \tanh(\mu)} \eta_i V_{in}(T_{in} - T_0) = c V_{in}(T_{in} - T_0) \]  

The linear theory does not give the absolute value for the coefficient \( c \), but it does provide
its dependence on the slope parameter $\delta$. The value of $c$ inferred from (4), making use of (3) is shown in Fig. 10b as a function of $\delta$ for the fastest growing wave. The value of $c$ was set arbitrarily to 1 for $\delta = 0$. The eddy heat flux is greatly reduced as the bottom slope increases relative to the isopycnal slope. For values typical of the Labrador Sea the growth rate is reduced approximately 80% relative to the flat bottom result. The dependence of $c$ on $\delta$ is well approximated by $e^{2\delta}$, as indicated by the dashed line in Fig. 10b. We can make use of the previous flat bottom research (Visbeck et al., 1996) to provide the constant of proportionality in (4) for $\delta = 0$ to be 0.025, so that the final eddy flux is related to the mean model parameters as

$$u'\theta' = 0.025e^{2\delta}V_{in}(T_{in} - T_0).$$

A similar functional relationship was tested in a series of numerical model calculations by Isachsen (2010). He found that the linear theory of Blumsack and Gierasch (1972) compared well with the nonlinear model results for the parameter range $-1 < \delta < 0$, which is the relevant range for convective basins.

It is assumed that the velocity of the inflowing water is in thermal wind balance with the density gradient between the boundary and the interior, and that the inflowing velocity is zero at the bottom, so that

$$V_{in} = \frac{\alpha g(T_{in} - T_0)H_{in}}{\rho_0 f_0 L}$$

where $\alpha$ is the thermal expansion coefficient, $g$ is gravitational acceleration, and $L$ is the width of the sloping topography.

An estimate of the temperature anomaly of the interior water mass is now derived by combining (1), (5), (6) to be
\[ (T_{in} - T_0) = \frac{1}{H_{in}} \left( \frac{A f_0 L Q}{\alpha g P C_p c} \right)^{1/2} \]  

This result is significantly different from the isolated deep convection scaling of Visbeck et al. (1996). Its power dependence on surface heat loss and horizontal scale are different, but more importantly the temperature anomaly now depends on the rotation rate, the topographic width, and the bottom slope (through \(c\)). If it is assumed that the width of the boundary current is the internal deformation radius, then the scaling of Visbeck et al. is recovered (Chapman, 1998). At high latitudes, the width of the sloping topographic is typically much wider than the internal deformation radius, so the distinction is important. If the boundary current is to be in thermal wind balance, then this expression also determines the baroclinic shear in the boundary current through (6). This implies that the strength of the cyclonic boundary current is controlled by heat loss in the interior of the basin and the ability of eddies to flux heat from the boundary current into the basin interior.

The prediction for the temperature of the convective waters in the interior of the marginal sea (7) was tested with a numerical model by Spall (2004). A series of calculations were carried out in which the basin radius, surface heat flux, Coriolis parameter, width and amplitude of the topographic slope, and depth of the basin were varied. The characteristics of the general circulation and identification of distinct water masses are similar in each of these calculations to the example in Fig. 4. The mean temperature of the water mass in the interior of the marginal sea diagnosed from each model run is compared to that predicted by (7) in Fig. 11a. The temperature anomaly of the convective waters varies from approximately 0.5°C to 2°C over all the calculations. The temperature predicted by the theory compares well with the model results over the whole range of parameters tested, with a correlation of 0.95 and a least square linear fit with slope 1.23. One of the main points from this result is that the temperature of the waters
formed in the basin interior varies considerably, even when subject to the same heat loss. It is clear that the geometry of the basin and the ability of eddies to form from the boundary current control the properties of the waters formed in the basin interior.

A second quantity of interest is the temperature of the waters exported from the marginal sea. This will, in general, be warmer than the convective waters in the basin interior and cooler than the inflowing water (as required to balance the heat loss in the marginal sea). The average temperature of the outflowing waters can be estimated by requiring that the net heat flux carried into the marginal sea by the mean boundary current balances the heat loss to the atmosphere (neglect any eddy heat fluxes through the strait), and that the net mass flux into the marginal sea is zero. The heat budget is

\[
T_{\text{in}} V_{\text{in}} H_{\text{in}} L - T_{\text{out}} V_{\text{out}} H_{\text{out}} L = \frac{AQ}{\rho_0 C_p},
\]

where the subscripts “in” and “out” refer to the inflowing and outflowing properties of the boundary current. The inflowing heat transport may be written, making use of (6) and (7) as

\[
T_{\text{in}} V_{\text{in}} H_{\text{in}} L = \frac{RQL}{2\rho_0 C_p L},
\]

where \( R \) is the basin radius.

The mass balance requires that \( V_{\text{in}} H_{\text{in}} L = V_{\text{out}} H_{\text{out}} L \). This may be combined with (8) and (9) to produce an estimate for the outflowing temperature, relative to the interior temperature, as

\[
T_{\text{out}} - T_0 = (T_{\text{in}} - T_0) (1 - \epsilon)
\]

The factor \( \epsilon = cP/L \), may be interpreted as the fraction of the inflowing waters that are fluxed into the interior by eddies (Spall, 2004). For steep topography, \( c \ll 1, \epsilon \ll 1 \) and most of the
water that flows into the marginal sea in the boundary current simply encircles the basin and flows back out of the marginal sea. For these cases, the temperature of the outflowing water is close to that of the inflowing water. However, for larger $cP/L$, eddies flux more heat into the basin interior and the temperature of the outflowing waters decreases. When $cP/L = 1$, all of the heat carried into the marginal sea in the boundary current is lost to the interior as it encircles the marginal sea. A slightly modified approach is required for $\epsilon > 1$ (see Spall, 2004). The average temperature of the outflowing waters diagnosed from the series of model calculations is compared to that predicted by (10) in Fig. 11b. Once again there is general agreement between the model and theory, with a correlation of 0.87 and slope of 1.35. For some calculations, the outflowing temperature is close to the inflowing temperature, while for others the outflowing temperature is close to the temperature of the convective water mass formed in the interior of the marginal sea.

The heat budgets of the interior of the marginal sea and the marginal sea as a whole have been used to predict the temperature of the convective and exported water masses. A related quantity of interest is the meridional overturning streamfunction, or net downwelling, in the marginal sea. The heat budgets alone do not inform us as to the means by which heat is transported into the basin. For example, the outflowing cold water could be in the upper part of the water column, in which case the heat transport would be achieved by a horizontal gyre and there would be very little downwelling in the basin. On the other hand, the outflowing waters could be found at depth, in which case the heat transport would be carried by an overturning gyre and the downwelling would be much larger. The maximum downwelling in the marginal sea can be estimated by consideration of the thickness of the outflowing water column $H_{out}$ relative to the inflowing water column $H_{in}$. The downwelling transport $W$ is given simply by the product of this change in thickness times the width of the boundary current $L$ and the velocity of the
boundary current $V_{out}$ as

$$W = (H_{out} - H_{in})V_{out}L \quad (11)$$

We can estimate the thickness of the outflowing water by making use of mass conservation in the marginal sea, the temperature of the outflowing water, and thermal wind balance to be

$$H_{out} = \frac{V_{in}H_{in}}{V_{out}} = \frac{H_{in}}{(1 - \epsilon)^{1/2}} \quad (12)$$

The downwelling is now derived to be

$$W = \frac{H_{in}}{\rho_0} \left( \frac{RLQ \alpha g}{2C_p c f_0} \right)^{1/2} [1 - (1 - \epsilon)^{1/2}] = [1 - (1 - \epsilon)^{1/2}]\Psi. \quad (13)$$

where $\Psi = V_{in}H_{in}L$ is the inflowing transport in the boundary current. The downwelling in the marginal sea is simply related to the inflowing transport by a factor of $[1 - (1 - \epsilon)^{1/2}]$. For $\epsilon \ll 1$, very little of the inflowing transport downwells, while for $\epsilon = 1$ all of the inflowing transport downwells within the basin. In the limit of $\epsilon \ll 1$

$$W \approx \frac{\epsilon}{2}\Psi. \quad (14)$$

In this limit very little of the inflowing transport downwells within the marginal sea. As $\epsilon$ increases, a larger fraction of the inflowing transport downwells. Since $\epsilon$ is a measure of the instability of the boundary current, it is clear that the net downwelling in the basin is closely related to the eddy fluxes from the boundary current into the interior.

The comparison between the maximum downwelling diagnosed from the model and that predicted by the theory (14) is shown in Fig. 11c. In general, the theory compares well with the model. It is clear from these results that the downwelling is not simply related to the heat loss.
in the basin since for most of these calculations the heat loss is the same yet the downwelling varies by almost an order of magnitude. In fact, the downwelling transport is uncorrelated with the density anomaly of the convective water mass (Fig. 11d). The downwelling in the basin is just what is required to balance the change in thermal wind transport from the inflowing to the outflowing boundary currents.

The relative importance of the horizontal and overturning gyres in the meridional transport of heat is also of interest. The ratio of the heat transport by the horizontal circulation to that by the overturning circulation can be estimated as

\[ \frac{V_{in}H_{in}L(T_{in} - T_{out})}{W(T_{in} - T_{out})} = \frac{1}{1 - (1 - \epsilon)^{1/2}} \approx \frac{2}{\epsilon} \]  

(15)

The final approximation is valid for \( \epsilon \ll 1 \). In this limit, which is relevant for the Labrador Sea, the heat transport is dominated by the horizontal circulation, not the overturning circulation. This is consistent with estimates from repeat hydrography by Pickart and Spall (2007).

c. moving further towards a more realistic configuration

Many marginal seas are separated from the open ocean by a topographic sill (e.g. the Nordic Seas, the Mediterranean Sea), while the previous configurations have only considered a sloping topography around the perimeter of the basin (which might be considered more relevant for the Labrador Sea). Another aspect of these previous calculations is that the heat loss to the atmosphere was specified, independent of the ocean circulation. This was done to demonstrate that differences in the circulation and exchange were due only to changes in the model configuration, not due to a change in the surface forcing. However, it is expected that the heat loss to the atmosphere will depend on the ocean circulation, in particular the sensible heat flux with the
atmosphere depends on the difference between the ocean temperature and the atmospheric temperature, such that a cooler ocean loses less heat to the atmosphere than a warm ocean. Finally, wind-forcing has been so far neglected, but this has been done largely to simplify the problem rather than having been justified by any *a-priori* scaling analysis. Each of these limitations are addressed in this final subsection.

The general approach is similar to what has already been discussed, and so only a brief overview will be given here. The model has been extended further to the south so that the “open ocean” is now large enough to support a wind- and buoyancy-driven circulation typical of a subpolar gyre. The domain extends $L_y = 2000$ km in the meridional direction and $L_x = 1000$ km in the zonal direction (Fig. 12). The topography consists of sloping bottom around most of the perimeter, except near the southern boundary. The vertical wall at $y = 0$ prohibits topographic contours from closing around the basin and limits the excitation of a basin-scale circulation around closed contours. A sill is placed at $y = 1200$ km, so that the marginal sea extends 800 km in the meridional direction. The surface heat flux is calculated by a linear relaxation of the upper model temperature towards the atmospheric temperature shown in Fig. 12 with a relaxation constant $\Gamma$ as

$$Q = \Gamma(T_{sst} - T_A)$$

(16)

where $\Gamma$ has the units $\text{W m}^{-2} \text{C}^{-1}$, and will be varied. The restoring temperature was chosen to represent a general warming at low latitudes and cooling at high latitudes with stronger cooling in the northwestern part of the basin. The model is forced by a wind stress as

$$\tau = i\tau_0^x \cos\left(\pi y/L_y\right) + j\tau_0^y \cos\left(\pi x/L_x\right).$$

(17)

The temperature is also restored towards a uniform stratification of $N^2 = 2 \times 10^{-6} \text{s}^{-2}$ with a
time scale of 30 days between \( y = 0 \) and \( y = 200 \) km.

The model has 5 km horizontal resolution and 20 levels in the vertical varying from 25 m in the upper 250 m to 250 m over the deepest 1250 m. The model is initialized at rest and run for a period of 30 years. The analysis is taken over the final 5 years of integration. The inclusion of a sill is similar to what was done by Iovino et al. (2008), and some of their results will be used in the following analysis. However, the heat flux parameterization allows us to better understand what controls the meridional heat transport across the sill, a basic quantity of importance for climate, and a quantity that was specified in the studies of Spall (2004) and Iovino et al. (2008). The addition of wind-forcing and a larger southern basin allow for a more complete representation of the circulation outside the marginal sea, and for the possibility of wind-driven exchange across the sill.

The mean temperature and horizontal velocity at the uppermost model level are shown in Fig. 13. For this calculation, the restoring time scale is \( \gamma = 60 \) days (or \( \Gamma = 20 \) W m\(^{-2}\)C\(^{-1}\), \( \gamma = h_1 \rho_0 C_p / \Gamma \), where \( h_1 \) is the thickness of the upper model level), the Coriolis parameter at the southern limit of the model domain is \( 1.4 \times 10^{-4} \) s\(^{-1}\), and \( \tau_0^x = 0.15 \) N m\(^{-2}\), \( \tau_0^y = 0 \). Although clearly much more idealized than the real ocean, this calculation does represent several key aspects of the observed circulation in the North Atlantic Ocean and Nordic Seas. The mean basin-scale circulation in both the marginal sea and the northern part of the open ocean is cyclonic. Warm water is advected northwards along the western boundary at low latitudes, crosses the basin to flow northwards along the eastern boundary. Upon reaching the latitude of the sill, some warm water continues northwards into the marginal sea and some turns towards the west along the southern flank of the ridge. The temperature of the water decreases along both the northward and westward pathways. The coldest waters are found in the interior of the marginal sea, while the water flowing out of the marginal sea along the western boundary
is colder than the inflowing waters along the eastern boundary.

Following the same general approach as in Section b, a heat balance in the interior of the marginal sea can be used to estimate the temperature of the convective water mass.

\[
PH\overline{u'T'} = \frac{A\Gamma(T_0 - T_A)}{\rho_0C_p}
\]  

(18)
The eddy heat flux into the interior is integrated only down to the sill depth \(H\) because the heat transport into the basin is confined to depths less than the sill depth (Iovino et al. 2008). The eddy heat flux is parameterized as previously in (5). Using thermal wind to relate the mean velocity to the temperature gradient between the boundary current and the interior, an estimate for the interior temperature is obtained.

\[
T_{in} - T_0 = \frac{\mu}{\epsilon}[(1 + 2\epsilon/\mu)^{1/2} - 1](T_{in} - T_A)
\]  

(19)

A new nondimensional parameter has been introduced

\[
\mu = \frac{A\Gamma f_0}{2\alpha g C_p H^2(T_{in} - T_A)}.
\]  

(20)
The physical interpretation of \(\mu\) will be discussed shortly.

13 model calculations have been carried out in which the sill depth, restoring constant, and Coriolis parameter have been varied. The diagnosed temperature of the convective waters (relative to the inflowing waters) is compared to that predicted by (19) in Fig. 14a. The atmospheric temperature \(T_A = 2.8\) used in (19) and (20) is the spatial average over the marginal sea. The central calculation in Fig. 13 is shown by the asterisk. The comparison is quite close over the entire range of calculations. The convective water temperature is most sensitive to the sill depth, with shallow sills producing the coldest waters. The convective water density is only weakly dependent on the Coriolis parameter and the restoring time scale.
Once the sea surface temperature is known, we can readily obtain an estimate for the heat loss to the atmosphere, which in steady state is also the meridional heat transport across the sill. Using (16), the heat loss is

\[ Q = A\Gamma [(1 + PL/A)(T_{in} - T_A) - (T_{in} - T_0)] \]  

(21)

The term containing \( PL/A \) accounts for heat loss directly from the boundary current to the atmosphere. The remaining, larger terms are driven by eddy fluxes into the interior. For simplicity, it is assumed that the temperature of the boundary current is \( T_{in} \), although this will produce a slight overestimate of the heat loss from the boundary current. The meridional heat flux across the sill diagnosed from the 13 model calculations is compared to the prediction (21) in Fig. 14b. While there is some scatter, in general the prediction compares well with the model results. The heat flux is most sensitive to variations in the restoring constant (triangles) and sill depth (squares). It is nearly independent of the Coriolis parameter (circles). The heat flux becomes very small for shallow sills (stars).

Note that four of these calculations do not have a sill. The presence of a sill influences the solution in two ways (Iovino et al. 2008). First, it limits the depth over which water can flow into the basin, and thus influences the strength of the velocity in the boundary current through thermal wind, and the depth to which eddies can flux heat into the interior. The deeper the sill, the larger the vertical extent over which one integrates the thermal wind relation, and the stronger the velocity in the boundary current for a given temperature gradient. The second influence is that the sill blocks some of the topographic contours from extending directly from the southern basin into the marginal sea. This effectively narrows the region over which the boundary current flows, increasing the vertical shear in the velocity for a given change in temperature from the boundary current into the interior because \( L \) decreases in (6). The same
approach works for calculations with no sill as long as the primary means of heat transport into
the northern basin is through the mean eastern boundary current over the sloping bottom.

The temperature in the interior of the marginal sea will fall between the temperature of the
inflowing water $T_{in}$ and the atmospheric temperature $T_A$. If the eddy flux of heat from the
boundary is sufficiently strong, the interior will be flooded with boundary current water before
its heat can be extracted by surface cooling. This situation is found in the Lofoten Basin of the
Nordic Seas, where the basin is filled by warm water of North Atlantic origin through lateral
spreading via eddies formed from the boundary current. However, if the eddy flux is relatively
weak, the atmospheric influence can overwhelm this eddy flux and strongly cool the ocean.
This is more similar to the interior of the Greenland Sea, where deep convection occurs and the
densest waters are formed within the Nordic Seas. In the former case there is a large heat loss
to the atmosphere while in the latter case the heat flux is relatively small. The primary reason
for this difference is that the topography along the eastern side of the Lofoten Basin is very
steep compared to that along the western side of the Greenland Sea, thus increasing the eddy
exchange through a smaller $L$ in $\epsilon$.

This transition between ocean influence and atmospheric influence is made clear if we con-
sider the nondimensional form of (19) by dividing both sides by the temperature scale $T_{in} - T_A$.
This nondimensional temperature anomaly of the convective waters depends only on the param-
eter $\mu/\epsilon$. A plot of the nondimensional temperature anomaly diagnosed from the model runs
as a function of $\mu/\epsilon$ is shown in Fig. 15a. The scaled temperature anomaly predicted by (19)
is given by the solid line. For $\mu/\epsilon \ll 1$ the sea surface temperature in the interior of the basin
is close to the inflowing temperature (ocean dominates), while for $\mu/\epsilon \gg 1$ the sea surface
temperature is close to the atmospheric temperature (atmosphere dominates). The transition
between ocean dominance and atmospheric dominance occurs around $\mu/\epsilon \approx 1$. 
So, what does the ratio $\mu/\epsilon$ represent? Its meaning becomes clear if we write it as the product of three terms:

$$\frac{\mu}{\epsilon} = \frac{\rho_0 f_0}{2\alpha g H^2 (T_{in} - T_A)} \frac{1}{\epsilon} \frac{\Gamma}{\rho_0 C_p} \quad (22)$$

The first term on the right-hand side is inversely proportional to the transport that would be carried in the boundary current if the interior of the marginal sea were at $T_A$. The second term projects this transport into an effective eddy transport into the basin interior (recall that $\epsilon$ is the fraction of the boundary current heat transport that is fluxed into the interior by eddies). The third is an effective “transport” from the marginal sea into the atmosphere based on the strength of the air-sea exchange coefficient $\Gamma$ (the units are $m^3 s^{-1}$). The ratio $\mu/\epsilon$ thus represents the transport from the ocean to the atmosphere compared to the transport from the boundary current into the basin interior. This is consistent with the general behavior found in Figs. 15.

This understanding can be used to obtain an approximate solution for $T_0$ as follows. An equilibrium solution is obtained when the heat transport to the atmosphere is balanced by the heat transport from the boundary current into the interior. We can approximate the heat transport to the atmosphere as being proportional to the third term on the right-hand side of (22) times the temperature difference $T_0 - T_A$. The heat transport from the boundary current into the interior is approximately given by the inverse of the first two terms in (22) times $T_{in} - T_0$. Balancing these two heat transports gives:

$$\frac{2\mu (T_0 - T_A)}{\epsilon (T_1 - T_0)} = 1 \quad (23)$$

This may be solved for the nondimensional temperature anomaly to be

$$\frac{(T_1 - T_0)}{(T_1 - T_A)} = \frac{2\mu/\epsilon}{1 + 2\mu/\epsilon} \quad (24)$$
This approximate solution reproduces the limits and transitions of the full theory reasonably well, but underpredicts the temperature anomaly for small $\mu/\epsilon$ (Fig. 15a). This is because the approximate solution assumes that the interior temperature is at $T_A$ for the boundary current transport, while for small $\mu/\epsilon$ it is warmer than $T_A$, resulting in a smaller transport in the boundary current and thus less eddy flux into the interior. The approximate solution for $T_0$ can also be used to obtain an approximate solution for the meridional heat transport across the sill, which is shown in Fig. 15b by the dashed line. The overall agreement between the approximate theory and the full theory is sufficiently close in both cases that it lends support to the simple interpretation of the parameter $\mu/\epsilon$ and the basic balance that determines the equilibrium temperature in the marginal sea.

The temperature of the outflowing waters can be calculated using the heat budget for the entire marginal sea, as was done in the previous section for the case with a specified heat flux. Following a similar procedure using the heat budget, thermal wind, and mass conservation, the nondimensional outflowing temperature anomaly, relative to the inflowing $T_{in} - T_{out}$ is estimated to be:

$$T_{in} - T_{out} = 2\mu\left[\left(\frac{1 + P_\epsilon L/A}{T_1 - T_0}\right) - 1\right](T_1 - T_A).$$

(25)

The temperature anomaly of the outflowing temperature calculated from the model runs is compared to that predicted by (25) in Fig. 16. The general trend is reproduced, but the theory underpredicts the temperature of the outflowing water slightly. Shallow sills produce the largest difference in temperature between inflowing and outflowing waters. This is because the mass transport in the boundary current is reduced for shallow sills, thus requiring a larger temperature anomaly to balance surface cooling. The outflowing water temperature depends on both $\mu$ and the ratio $\mu/\epsilon$, so a simple comparison of the model results with a single nondimensional number
is not possible.

It is also useful to calculate the outflowing temperature relative to the convective water temperature.

\[
(T_{\text{out}} - T_0) = \Theta = (T_{\text{in}} - T_0) - 2\mu(T_1 - T_A)[\frac{T_1 - T_A}{T_{\text{in}} - T_0} (1 + PL/A) - (T_1 - T_0)] \tag{26}
\]

The ratio \(\Theta = (T_{\text{out}} - T_0)/(T_{\text{in}} - T_0)\) is the fraction of outflowing water that is composed of inflowing water. For \(\mu \ll 1\) the outflowing water is close to the inflowing water temperature, while for \(\mu \gg 1\) the outflowing water temperature approaches the atmospheric temperature. The ratio \(\Theta\) can be used to obtain an estimate of the total downwelling within the marginal sea. By requiring that the net mass transport into the marginal sea is zero, and assuming that the inflowing and outflowing currents are in thermal wind balance with the temperature gradients predicted by (19) and (26), one can obtain an estimate for the maximum downwelling in the marginal sea as

\[
W = \Psi(\Theta^{1/2} - \Theta) \tag{27}
\]

For \(\Theta \rightarrow 0\), the temperature of the outflowing water approaches the temperature of the convective water mass, while for \(\Theta \rightarrow 1\) the outflowing water is composed of inflowing water. The transport in the boundary current is given by \(\Psi\), so the downwelling is directly related to the inflowing transport through \(\Theta\). The maximum overturning rate at the location of the sill diagnosed from the model is compared to that predicted by (27) in Fig. 17. The theory compares well with the model results over the entire range of parameter space. The strength of the overturning varies between 0.25 Sv and 4.12 Sv. Unlike the temperature of the water masses produced, the overturning is very sensitive to the Coriolis parameter (circles) because of its dependence on
which, through thermal wind, depends inversely on $f_0$. It is also sensitive to the restoring constant (triangles) and the sill depth (squares).

d. influence of wind-forcing

The theory derived above does not take specific account of wind-forcing. It simply defines the characteristics of the boundary current that would be required to balance surface heat loss with the assumption that all of the heat is provided through instability of the boundary current. Wind-forcing was not included in the previous marginal sea studies of Spall (2003, 2004), Iovino et al. (2008), Walin et al. (2004), or Straneo (2006). In the calculations of Straneo, an idealized buoyancy-forced only model was able to reproduce the general characteristics of the observed seasonal and interannual variability in the Labrador Sea, suggesting that wind effects are secondary.

We can expect wind-forcing to influence the exchange across the sill in several ways. First, if there is a zonal component to the wind there will be a meridional Ekman transport across the sill with magnitude (per unit zonal distance)

$$ v_E = -\frac{\tau_0}{\rho_0 f_0}. $$

This transport will be to the north for winds towards the west. For parameters typical of the present model calculations, and the subpolar North Atlantic and Nordic Seas, $v_E \approx 0.5 \text{m}^2 \text{s}^{-1}$. The meridional transport is calculated by multiplying this by the width of the basin, which is $O(10^6 \text{ m})$, gives a total transport of $O(0.5 \text{ Sv})$. This is much less than the transport found in the northward flowing boundary current in the present model calculations (approximately 9 Sv) or between the North Atlantic and the Nordic Seas, and suggests that this effect is small.
The heat transport carried by this Ekman transport is calculated from the difference between the inflowing and outflowing temperatures. The sea surface temperature just south of the sill is cooler than the inflowing waters found along the eastern boundary, so the heat transport carried by this wind-driven flow is similarly negligible compared to the heat transport carried in the boundary current.

Another way in which the wind-forcing can drive flow across the sill is through the Sverdrup-driven interior flow. Cyclonic wind stress curl will force upwelling into the Ekman layer and drive a poleward flow on a beta-plane. However, because the stratification is relatively weak at these latitudes, the flow feels the bottom topography. Realistic configurations of the sill provide such a large effective beta through the bottom slope that the resulting meridional transport is negligible.

The third way in which wind-forcing can drive an exchange is if there is a wind stress parallel to the eastern or western boundary. For a northward wind adjacent to the eastern boundary, the Ekman transport will force a convergent flow into the boundary, which will result in an increase in sea surface height. The resulting pressure gradient will in turn drive a northward flow along the boundary. If the topographic contours extend from the southern domain directly into the marginal sea, then this will result in a wind-driven exchange.

We can estimate the strength of such a boundary current, in the simplest case, if we consider a northward wind of strength $\tau^y$ north of a given latitude, specified arbitrarily as $y = 0$, and zero south of that latitude. Nonlinearity, stratification, and friction are neglected. While obviously very idealized, this approach allows us to derive a simple expression for the transport across the sill. Because the forcing is zero south of $y = 0$, the strength of the meridional flow will be a function of latitude (e.g. Allen, 1976). The strength of the steady meridional velocity can be calculated by integrating the wind stress forcing along characteristics from $y = 0$ to latitude $y$. 26
The steady solution is set up by a series of coastal trapped waves, each higher wave moving at a slower phase speed and contributing less to the final solution. An order of magnitude estimate of the velocity in the boundary current can be obtained by considering the first mode only. Following Allen (1976), the transport carried by the boundary current is estimated to be

\[ V = \frac{yL\tau^y}{\rho_0 c_0} \]  

For the central case with a 1000 m deep sill, \( L = 70 \) km, \( y = 1200 \) km, and \( \tau^y = 0.075 \) N m\(^{-2}\). If we take a simple approximation for the barotropic shelf wave speed to be \( c_0 = f_0L \approx 10 \) m s\(^{-1}\) (Brink, 1982), this results in 0.62 Sv of wind-driven transport across the sill. This is an order of magnitude less than the mean transport with wind- and buoyancy-forcing, and suggests that wind effects will be small.

In order to test the effect of the Sverdrup and Ekman driven exchange across the sill, a series of model calculations have been run in which the zonal wind stress \( \tau_0^x \) with sill depths of 300m, 600 m, 1000 m, and 2000 m. Recall that the calculations in the previous section all had \( \tau_0^x = 0.15 \) N m\(^{-2}\). The resulting temperature of the convective water mass, meridional heat flux across the sill, and the meridional overturning at the sill are shown in Fig. 18. On the vertical axis is the value in the absence of wind forcing. Each symbol represents a diagnosed quantity for a case with wind-forcing (asterisks: \( \tau_0^x = 0.15 \) N m\(^{-2}\), circles: \( \tau_0^x = 0.30 \) N m\(^{-2}\)). If the data points were to fall on the diagonal line, then the results would be the same as the case with no wind. It is found that the properties of the exchange across the sill are very nearly the same for all cases and for each sill depth. The influences of the Ekman-driven transport and the Sverdrup transport on the exchange are negligible.

Calculations with a meridional wind of strength \( \tau_0^y = 0.075, 0.15 \) N m\(^{-2}\) are shown by the right facing and left facing triangles, respectively. Once again the results are very similar to the
case with no wind. The transport along the eastern boundary into the marginal sea increases from 8.6 Sv to 9.2 Sv to 9.7 Sv for $\tau_0 = 0.0, 0.075, 0.15 \, \text{N m}^{-2}$. This increase in transport of approximately 0.5 Sv for increasing wind stress of 0.075 N m$^{-2}$ is consistent with the above simple theory (29) for the wind-driven boundary current, and with the relatively minor influence of meridional winds on the exchange across the sill.

3. dynamics of downwelling

A quantity of much interest for the climate system is the meridional overturning circulation. Considerable attention has been paid to where these dense waters upwell and return to the surface. The two dominant regimes are diapycnal mixing in the deep ocean (Polzin et al, 1997) and wind-driven upwelling in the Southern Ocean (Toggweiler and Samuels, 1995). There has also been a lot of interest in where the poleward flowing waters in the upper limb of the thermohaline circulation become more dense, or downwell in density space (Talley, 2003). Until fairly recently, little has been known about where or how these upper layer waters sink to intermediate and deep depths. It should be evident from the previous section that the net downwelling attained in marginal seas is closely related to the surface buoyancy-forcing, but it should also be clear that the dynamics of the flow also influence the net downwelling rate. In this section, the dynamical balances in several different regimes that can support net downwelling motions in buoyancy-forced basins are discussed.

a. dissipative, stratified flows

The continuously stratified linear model described by Barcilon and Pedlosky (1967, hereafter BP67) defines two viscous boundary layers that are required to satisfy boundary conditions
for the normal and tangential components of the velocity and thermal insulation. The wider of
the two layers scales as $\sigma^{1/2} L_d$, where $\sigma = A_h/A_T$ is the horizontal Prandtl number and $L_d$
is the internal deformation radius. The width of this theoretical boundary layer is controlled
by coupled balances in the density and vorticity equations. Horizontal diffusion of density is
balanced by vertical advection of the mean stratification. This vertical advection gives rise to
stretching of planetary vorticity in the vorticity equation, which is balanced by horizontal dif-
fusion of relative vorticity into the boundary. This interplay gives rise to the dependence on
the horizontal Prandtl number. Thus, subgridscale mixing plays a key role in the dynamics. It
is demonstrated below that this layer plays the primary role in the net vertical mass transport
even in a time-dependent, nonlinear numerical model. There also exists a thinner, nonhydro-
static layer that does not contribute significantly to the net vertical mass transport, and will not
be discussed further here (see Pedlosky, Chapter XX).

The net downwelling in the flat bottom primitive equation calculation discussed in the pre-
vious section was concentrated adjacent to the boundary even though the surface cooling was
localized to the interior of the basin (Fig. 3). The basic balances operating in this downwelling
region can be exposed using a simple two-layer analytic model. Following Spall (2003), the
linear vorticity equation with lateral viscosity may be written for each layer as

$$(-1)^n \frac{f_0 w}{h_n} = -A_h \nabla^2 \zeta_n$$

where $f_0$ is the constant Coriolis parameter, $A_h$ is the lateral viscosity, $\zeta_n = \partial v_n/\partial x - \partial u_n/\partial y$
is the relative vorticity, $v_n$ and $u_n$ are the horizontal velocities, $h_n$ is the layer thickness, and
the subscript $n$ denotes the layer. The vertical velocity $w$ is defined at the layer interface. For
simplicity, it is assumed that the unforced layer thicknesses are $h_1 = h_2 = H$.

The model is forced by restoring the upper layer thickness $h_1$ towards a specified thickness
with time scale $\gamma$. This is intended to represent the advection of cold upper layer water of thickness $H$ formed by convection in the basin interior towards the boundaries by mesoscale eddies. For $h_1 > H$, this causes the upper layer to become thinner and the lower layer to become thicker. Because the lower layer is of higher density, this represents cooling and a diapycnal mass flux.

It is assumed that the meridional velocity is in geostrophic balance so that $v_1 - v_2 = P_x/f_0$. We also expect a boundary layer structure so that $v_x \gg u_y$. A single equation for the perturbation pressure can be derived by subtracting the layer 2 vorticity equation from the layer 1 vorticity equation.

$$P_{xxxx} + \frac{2f_0^2}{g' H \gamma A_h} P = 0$$

(31)

The vertical stretching of planetary vorticity has been linearized with respect to layer thickness, so $H$ is used instead of $h_n$ and the cross isopycnal velocity is assumed to be entirely vertical.

The equations may be nondimensionalized using the following scaling:

$$P \propto V f_0 L_d$$

$$x, y \propto L_d = \left(\frac{g' H}{f_0}\right)^{1/2}$$

(32)

where $V$ is a characteristic horizontal velocity scale. The nondimensional pressure equation is now

$$P_{xxxx} + \left(\frac{L_d}{\delta}\right)^2 P = 0$$

(33)

where $\delta = (A_h \gamma)^{1/2}$. It is assumed that the solution can be separated into an exponential in the offshore direction and an unknown function in the meridional direction,

$$P = F(y)e^{ikx}$$

(34)
Substitution of (34) into (33) yields two roots that support bounded solutions for an eastern boundary, \( k_1 \) and \( k_2 \).

\[
k_{1,2} = \sqrt{2}/2(L_d/\delta)^{1/2}(\pm 1 - i)
\]

(35)

so that the dimensional boundary layer width is \( \Delta = (L_d \delta)^{1/2} \).

The pressure is now written as the sum of two boundary layers, each with an unknown meridional dependence. The no-slip boundary condition and the meridional momentum equation can be used to solve for the meridional functions (see Spall, 2003, for details). The full solution for the perturbation pressure is now written as

\[
P = \frac{k_2}{k_2 - k_1} e^{-(L_d/\delta)^{1.5}Ey}(e^{ik_1 x} - \frac{k_1}{k_2} e^{ik_2 x})P_0
\]

(36)

where \( P_0 \) is the amplitude at \( y = 0 \). The pressure anomaly decays in the offshore direction with scales \( k_1^{-1} \) and \( k_2^{-1} \). The pressure anomaly also decays exponentially in the along-boundary direction with decay scale inversely proportional to the Ekman number \( E = \sqrt{2}A_h/f_0L_d^2 \).

The boundary layer width depends on \( \gamma \) through \( \delta \). The time scale \( \gamma \) depends on how temperature anomalies are eroded near the boundaries. If temperature is forced directly by heat loss to the atmosphere, then this time scale would depend on the air-sea exchange. It is assumed here that the temperature anomalies are eroded by small-scale diffusion, for simplicity here assumed to be represented by horizontal Laplacian diffusion with coefficient \( A_T \). For this mixing, the time scale is then \( \gamma = \Delta^2/A_T \), and the dimensional boundary layer width is

\[
\Delta = L_d\sigma^{1/2}
\]

(37)

where \( \sigma = A_h/A_T \) is the horizontal Prandtl number. This is the same horizontal length scale that arises in the continuously stratified analytic model of BP67, and was identified as the boundary
layer that supports most of the net buoyancy-forced vertical motion (see also the more detailed discussion by Pedlosky, this volume). This result was supported in the laboratory experiments of Whitehead and Pedlosky (2000).

The previous studies by BP67, Whitehead and Pedlosky (2000), and Pedlosky (2003) used an axisymmetric basin so that there was no along-boundary variation in the pressure field. In the present study, the pressure decays exponentially along the boundary (Eqn. (36)) with a dimensional e-folding distance of

$$
\chi = \frac{L_d \sigma^{1.5}}{E} = \frac{\Delta}{E_T}
$$

(38)

where $E_T = \sqrt{2} A_T / f_0 L_d^2$ is a thermal Ekman number.

These two basic descriptors of the boundary layer structure, the width of the downwelling region and the along-boundary decay scale, were diagnosed in a series of primitive equation model calculations in the flat bottom domain, as in Fig. 2, by Spall (2003). The Coriolis parameter, horizontal viscosity, horizontal diffusion, open ocean stratification, and basin depth were varied. In all cases, the basic circulation is similar to that shown in Figs. 2 and 3, although the distance the boundary current penetrates into the basin varies, as suggested by (38). The width of the boundary layer was diagnosed as that distance from the boundary at which the azimuthally averaged vertical velocity was reduced to 10% of its maximum value. This is compared with the theoretical scaling (37) in Fig. 19a. The agreement between model and theory is fairly good, with the boundary layer varying in width from less than 10 km to almost 40 km. The along-boundary decay scale was diagnosed from the model results as the distance at which the temperature anomaly on the boundary was reduced by a factor of $e^{-1}$. The theory predicts the general trends found in the model result reasonably well (Fig. 19b). The distance the boundary layer penetrates into the basin varies by an order of magnitude, from 100 km to
1000 km, for the same magnitude of surface forcing.

While this model calculation is quite idealized, it demonstrates a couple very useful points. As noted in Section 2, the vertical velocity is located adjacent to the lateral boundary. The present analysis demonstrates that this is due to the vorticity constraints on the system. The stretching of planetary vorticity that results from the net vertical motion needs to be balanced in the vorticity equation. This is efficiently accomplished near the boundary because relative vorticity can be removed through lateral viscosity. The characteristics of this downwelling region depend on both the physical length scale of the internal deformation radius and also on the numerical subgridscale parameterization.

\section*{b. weak dissipation, stratified flows}

While the boundary layer theory for downwelling in stratified flows by BP67, and its applications in Spall (2003) and Pedlosky (2003), depends on the diapycnal mixing of density due to horizontal diffusion, diapycnal mixing in the ocean is generally weak. The aim of this section is to explore downwelling in a convective basin with very small explicit diffusion of density. For a more detailed discussion, the reader is referred to Spall (2010a).

The MITgcm is configured in an elongated basin subject to cooling at the surface, which is connected to a smaller rectangular region through a strait (Fig. 20). The domain has topography along the perimeter that slopes exponentially from the surface down to the bottom depth of 3000 m with a horizontal e-folding scale that varies from 40 km over most of the basin to 10 km along the eastern boundary between $y=400$ km and $y=700$ km. The model is forced by applying $50 \text{ W m}^{-2}$ uniform cooling at the surface over the northern basin and by restoring towards a temperature profile with uniform vertical stratification of $N^2 = (g/\rho_0)\partial \rho / \partial z = 10^{-6} \text{ s}^{-2}$ in the southern rectangular region. This gives a first baroclinic deformation radius, based on the full
ocean depth, of \( L_d = NH/f_0 = 30 \text{ km} \).

The vertical grid spacing is 100 m over the full depth of 3000 m (30 levels). The model has variable horizontal grid spacing, as indicated in Fig. 20. The grid spacing in the region along the eastern boundary is 1 km. The model was first spun up for a period of 20 years with a slightly lower resolution grid (the region of 1 km grid spacing was 2 km). The temperature and velocity fields at the end of that calculation were then interpolated onto the finer grid to initialize the final 200 day calculation used in the analysis below.

The model incorporates second-order vertical viscosity and diffusivity with coefficients \( 10^{-5} \text{ m}^2 \text{ s}^{-1} \). The vertical diffusion is increased to \( 1000 \text{ m}^2 \text{ s}^{-1} \) for statically unstable conditions in order to represent vertical convection. Horizontal viscosity is parameterized as a second order operator with the coefficient determined by a Smagorinsky (1963) closure as

\[
A_h = \left( \nu_s/\pi \right)^2 L^2 \left[ (u_x - v_y)^2 + (u_y + v_x)^2 \right]^{1/2},
\]

where \( \nu_s = 2.5 \) is a nondimensional coefficient, \( L \) is the grid spacing, and \( u \) and \( v \) are the horizontal velocities (subscripts indicate partial differentiation). Temperature is advected with a third-order direct space/time flux-limiting scheme. There is no explicit horizontal diffusion of temperature.

The basin-scale circulation consists of a warm, cyclonic boundary current that enters the cooling basin along the eastern side of the strait and exits the basin along the western side of the strait, similar to the circulation discussed in Section b. The boundary current accelerates over the region of steep topography, which results in enhanced instability and eddy formation compared to the rest of the domain where the topographic slope is weaker (Katsman et al., 2004; Wolfe and Cenedese, 2006; Bracco et al., 2008; Spall, 2010a,b). This is reflected in the eddy kinetic energy (Fig. 20), which is a maximum in the region of steep topography and has high values extending into the basin interior as a result of eddy shedding from the boundary current.

Similar regions of enhanced eddy kinetic energy adjacent to steep topography are found along
the eastern boundaries of the Labrador Sea (Prater, 2002) and Lofoten Basin in the Nordic Seas (Poulain et al., 1996).

Vertical sections of the meridional average of the 200 day time-mean meridional velocity and temperature within the white box in Fig. 20 are shown in Fig. 21. The mean boundary current is approximately 30 km wide with a maximum velocity just over 30 cm s\(^{-1}\). The mean horizontal velocity is essentially parallel to the boundary, there is very little mean flow between the boundary region and the interior.

The time-mean Eulerian vertical velocity is small everywhere in the cooling basin except along the region of steep topography and eddy activity. A meridional average of the vertical velocity in the analysis box is shown in Fig. 21b. Over 95% of the total downwelling in the basin is concentrated in this relatively narrow band of O(20 km) width along the eastern boundary, precisely where the eddies are formed. The maximum basin-integrated downwelling transport of \(1.8 \times 10^6 \text{ m}^3 \text{s}^{-1}\) is located at 1100 m depth.

The importance of eddy fluxes in the region of downwelling is indicated by the flux divergence of the advective terms in the temperature tendency equation shown in Fig. 22. The advective terms essentially balance at all depths, indicating that local surface cooling does not penetrate below 100 m in the boundary current and that diffusive effects are small. The mean advection is acting to warm the region throughout the water column. This represents the inflow of warm water in the cyclonic boundary current. Anticyclonic eddies transport warm water from the boundary current into the interior and act to cool the region at all depths (as shown in Fig. 25). Similar warm anticyclones are observed to be formed along the eastern boundaries of the Labrador Sea and the Lofoten Basin (Lilly and Rhines, 2002; Poulain et al., 1996). The vertical eddy fluxes cool the deep ocean and warm the upper ocean. This is consistent with baroclinic instability, which releases potential energy by upwelling warm water and down-
welling cool water (analysis of the energy conversion terms confirms baroclinic instability is active).

The mean temperature and northward transport between 400 km and the boundary is shown in Fig. 23a. The temperature in the boundary current decreases at all depths as one moves northward, as expected since the boundary current loses heat as a result of eddy exchange with the interior. The mean along boundary transport deepens, but does not decrease, as the boundary current flows northward as a result of the downward mean vertical velocity. If eddies were not important for the heat budget, this would require an unrealistically large diapycnal mixing coefficient of $O(10^{-2} \text{m}^2 \text{s}^{-1})$ to balance this cross-isotherm mean advection.

The influence of eddies on density advection can be represented in compact form by considering the Transformed Eulerian Mean formulation (e.g. Andrews and McIntyre, 1978). This representation includes the effects of eddy fluxes, but written in such a way that they appear as an advection acting on the mean density field. If it is assumed that the eddy advection of density is adiabatic, then these eddy-induced cross-front and vertical velocities may be written in the form of a streamfunction (where subscripts indicate partial differentiation) as

$$
\ddot{u} = -\psi_z \quad \quad \ddot{w} = \psi_x.
$$

There are numerous choices for the streamfunction $\psi$ as reviewed, for example, by Vallis (2006, pp. 304-314). For the primitive equations, a convenient choice is

$$
\psi = -\frac{w'\bar{\rho}' \bar{p}_z - w'\bar{\rho}' \bar{p}_x}{\bar{\rho}_x^2 + \bar{\rho}_z^2}. \quad (40)
$$

where the overbar indicates a time average and the primed variables are deviations from the mean. The eddy flux is dominated by the zonal component, $(v'\bar{\rho}')_y \ll (w'\bar{\rho}')_x$ and $\bar{\rho}_{xy} \ll \bar{\rho}_{xx}$, so the meridional eddy flux term is neglected.
The streamfunction in (40) was calculated at each grid point in the analysis box and then averaged in the meridional direction to yield a two-dimensional depiction of the eddy-induced velocity. The time averaged terms are only weakly dependent on latitude, so this averaging is used to reduce the influence of synoptic variability in $\psi$. The effective eddy-induced vertical velocity $\tilde{w}$ calculated from this averaged streamfunction via (39) is shown in Fig. 23b. It is nearly equal in magnitude and opposite in sign to the Eulerian mean vertical velocity in Fig. 21b. Such an upward eddy heat flux is consistent with Fig. 22 and baroclinic instability. An along-boundary transport streamfunction was calculated using the zonaly averaged residual vertical velocity (the sum of sections in Fig. 21b and Fig. 23b). This results in upwelling that is in general agreement with the rise of the mean isopycnals (Fig. 23c), indicating that the flow is nearly adiabatic.

The question remains as to what balances the mean Eulerian vertical velocity in the vertical component of the relative vorticity equation.

$$\zeta_t = -\overline{\vec{v}} \cdot \nabla \zeta - \nabla w_z - \nabla w \cdot \vec{v}_z + \zeta w_z - \beta v + f w_z + A \nabla^2 \zeta + K \zeta_{zz} \approx 0$$

(41)

advection + stretching + dissipation

The balance in the boundary layer theory of BP67 is between the last two terms, stretching and dissipation. All three terms have been calculated from the model output and averaged in the meridional direction to produce zonal sections (Fig. 24). The advection term includes all advective effects, including horizontal advection of relative vorticity, vertical advection of relative vorticity, tilting of horizontal vorticity, and advection of planetary vorticity.

The strong mean Eulerian downwelling results in stretching of planetary vorticity, $fw_z$, that tends to increase the upper ocean relative vorticity in the region of maximum downwelling. This is balanced to a large degree by the an advective export of positive vorticity (primarily
the horizontal eddy flux divergence of relative vorticity). The majority of the relative vorticity flux is towards the boundary, but there is also an export of positive relative vorticity into the interior (not shown). The flux towards the boundary goes to zero at the boundary because of the no-normal flow boundary condition, giving rise to a flux convergence (the positive region near the boundary in the upper 700 m) that is primarily balanced by dissipation. Thus, unlike the theory of BP67, the region of dissipation is offset from the region of downwelling, but they are connected through the lateral eddy flux term. The positive relative vorticity flux into the basin interior is somewhat surprising since the dominant form of eddies exported from the boundary current are anticyclonic, and points to the importance of sub-mesoscale filaments of positive relative vorticity in the eddy field (Fig. 25). The primary balance, when integrated over the whole downwelling region, is between stretching and dissipation, similar to the BP67 balance. However, the positive relative vorticity flux into the interior balances up to 20% of the stretching (Fig. 24d).

c. weakly stratified flows

The idealized modeling study of Spall and Pickart (2001) found that essentially all of the downwelling at high latitudes in a low resolution, basin-scale model occurred within the mixed layer adjacent to the boundary. Their model was non-eddy permitting and had relatively weak diapycnal mixing, so there was no means to support downwelling in the stratified part of the water column. They argued that the magnitude of the downwelling was determined by the mixed layer depth and density gradient along the boundary through thermal wind balance and a no-normal flow condition through the wall. This weakly stratified regime differs significantly from the previous theoretical analysis, which assumes that the stratification is significant. This motivates a more detailed analysis of the weakly stratified regime, and the discussion here
follows closely from Spall (2008). The primary approach is numerical, although a scaling is used to identify the key boundary layer that supports the downwelling motion. For a more detailed discussion of this boundary layer and its connection to the interior flow, see Pedlosky (2009) or Chapter XX in this volume.

A high resolution numerical model is used to calculate the vertical motions within the mixed layer forced by cooling of a boundary current. The MITgcm is run with nonhydrostatic, Boussinesq physics. The model domain is a channel of width 20 km, length 48 km, and depth 500 m. The model horizontal grid spacing is 100 m and the vertical grid spacing is 10 m. Subgridscale mixing of momentum and temperature are parameterized by a horizontal Laplacian mixing with coefficients of $1 \text{ m}^2 \text{s}^{-1}$ and vertical mixing coefficients of $10^{-5} \text{ m}^2 \text{s}^{-1}$.

The initial stratification is uniform with $N^2 = 4.8 \times 10^{-6} \text{ s}^{-2}$. The model is forced with a specified inflowing velocity in geostrophic balance with the density field and cooled at the surface with a uniform heat flux of 500 W m$^{-2}$ (Fig. 26). The Coriolis parameter is $f_0 = 10^{-4} \text{ s}^{-1}$ and uniform. The inflowing velocity has a maximum value of 30 cm s$^{-1}$ at the surface on the southern boundary and decreases linearly to zero at 500 m depth and at the northern side of the domain\(^1\). The model is initialized with this velocity field and a geostrophically balanced density field and sea surface height. The inflow conditions are steady in time and the outflow boundary conditions for temperature, normal velocity, and tangential velocity are determined by an Orlanski radiation condition (Orlanski, 1976), see the MITgcm web page for details of the numerical implementation (http://mitgcm.org/r2_web_testing/latest/home_page/frontpage.html). The northern and southern lateral boundary conditions are no normal flow, no-slip, and no normal heat flux.

\(^1\)The calculations are all on an f-plane but, for convenience, the direction of flow will be considered towards the east and the offshore side of the boundary current will be towards the north.
The temperature and inflowing velocity at 45 m depth, averaged between days 2 and 6, are shown in Fig. 26. The temperature change across the boundary current at the inflow boundary is approximately $0.45^\circ$ C. As a result of the surface cooling, the horizontal temperature gradient at the outflowing boundary has been reduced to approximately $0.25^\circ$ C. The temperature at the offshore side of the current decreases only slightly over the length of the channel, while the temperature of the onshore side of the boundary current decreases by over $0.2^\circ$ C. The rate of cooling is fastest near the inflow boundary (once cooling has penetrated to this depth, approximately 5 km downstream from the inflow) and decreases downstream. This is because the uniform cooling is distributed over an ever increasing vertical extent as the mixed layer increases from zero at the inflowing boundary to approximately 200 m at the outflowing boundary. The decrease in lateral temperature gradient from inflow to outflow implies a more barotropic boundary current since, through geostrophy, the vertical shear in the along-channel velocity is related to the lateral density gradient. This implies that there has been a redistribution of mass in the vertical such that the uppermost velocity has decreased and the velocity at some deeper level has increased, requiring a net downwelling within the domain.

Vertical sections of zonal velocity, meridional velocity, temperature, and a meridional/vertical plane streamfunction are plotted in Fig. 27. These quantities were averaged between days 2 and 6, and between $x = 20$ km and $x = 30$ km. The mixed layer depth, based on a change in temperature of $0.05^\circ$ C relative to the sea surface temperature, is indicated by the white line. The most evident change in the zonal velocity is the development of a no-slip boundary layer within approximately 2 km of the southern boundary throughout the depth of the domain. The meridional velocity is dominated by two cells, both within the mixed layer (Fig. 27b). The interior (away from the southern boundary) is characterized by northward flow of $O(1$ cm s$^{-1}$) in the upper portion of the mixed layer and southward flow of similar magnitude in the lower
mixed layer. The sense of this cross-channel circulation is to restratify the mixed layer. Near the southern boundary, the meridional flow is $O(10 \text{ cm s}^{-1})$ and towards the boundary in the upper mixed layer and away from the boundary in the lower mixed layer. The temperature field shows weak stratification within the mixed layer and a temperature inversion very close to the surface (Fig. 27c).

A meridional/depth transport streamfunction has been calculated by integrating the vertical velocity between $x = 20 \text{ km}$ and $x = 30 \text{ km}$, and then integrating from the southern boundary to the northern boundary at each depth, with $\psi = 0$ at $y = 0$, so that the streamfunction has units of $m^3 \text{s}^{-1}$. Although the flow in this plane is divergent ($\partial u / \partial x \neq 0$) the streamfunction presented here is a useful way to visualize the ageostrophic motions that characterize the vertical velocity in the interior because $(v_a, w) = (\psi_z, -\psi_y)$, where $v_a$ is the ageostrophic meridional velocity. The vertical motions characterize only the ageostrophic meridional velocity because the geostrophic flow is horizontally nondivergent. The vertical motions are contained primarily in the mixed layer (Fig. 27d). There is a very strong cell adjacent to the southern wall that extends from the surface down to the bottom of the mixed layer, with intense downwelling adjacent to the boundary and weaker upwelling spread over 2-3 km near the wall. The maximum vertical velocities are several cm s$^{-1}$ right next to the wall. There is a weaker cell near the northern wall with upwelling next to the boundary and downwelling just offshore. Within the basin interior the overturning cell is composed largely of weak, closed cells in the direction of restratification, with upwelling of warmer water and downwelling of colder water.

The vertical velocities are locally much larger than the net vertical motion. There are several different processes active, giving rise to different vertical velocities in different parts of the domain. The weak overturning in the mixed layer away from the boundary is opposite to what would be expected for a geostrophically balanced flow, in which the upper mixed layer flow
would be towards the southern boundary and the deep mixed layer would be away from the boundary (Spall and Pickart, 2001). Such a geostrophic flow is found if the viscosity and diffusivity are increased by a factor of 5 (discussed further below). The restratifying flow is due to symmetric instabilities, which are driven by buoyancy-loss at the surface.

The strong overturning adjacent to the southern boundary is a direct result of the lateral shear found in the no-slip boundary layer. Far from the wall, the relative vorticity is much smaller than $f$ and the along-channel pressure gradient is balanced largely by a weak cross-channel geostrophic flow. However, within a few km of the boundary, the horizontal shear of the along-channel flow is sufficiently large that the relative vorticity is $O(-f_0)$, so that the absolute vorticity becomes much smaller than $f_0$ (but not less than 0). The nonlinear momentum balance, appropriate for this region, becomes $(f_0 - u_y)v \approx P_x$. The pressure gradient is maintained by the surface cooling. As the absolute vorticity becomes small, the meridional velocity must increase in order to balance the pressure gradient.

The quantity of interest is the overall net vertical motion, indicated by the average vertical transport as a function of depth calculated between days 2 and 6 and between $x = 20$ km and $x = 30$ km and $y = 0$ and $y = 20$ km, as shown in Fig. 28. There is downwelling over most of the upper ocean, increasing from zero at the surface to a maximum of almost 6000 m$^3$ s$^{-1}$ at 100 m depth. Most of the net vertical motion is contained in the mixed layer (the average mixed layer depth over this region is indicated on the figure by the dashed line).

*idealized cases*

The essential feature of the previous calculation that results in a net downwelling is the decrease in mixed layer density change across the current in the downstream direction. This results from having weaker horizontal advection and a deeper mixed layer on the offshore side of the front than near the southern boundary while having a uniform heat loss at the surface.
The deeper mixed layer occurs because the horizontal velocity is weaker on the offshore side of
the front, thus resulting in deeper mixing for the same heat loss. However, a similar density field
arises if the horizontal velocity is uniform and the heat loss is greater near the southern bound-
dary than it is offshore. Imposing a spatially variable surface cooling with a spatially uniform
velocity and stratification allows for other configurations that, while perhaps less realistic than
the previous calculation, aid in identifying the important processes that control the net vertical
motion resulting from cooling.

Consider first the case of an inflow with uniform horizontal and vertical stratification and
a surface cooling that linearly decreases from 1000 W m\(^{-2}\) at the southern boundary to zero
at the northern boundary (same total heat loss as in the previous case). The geostrophically
balanced initial condition and inflow have a zonal velocity that is uniformly sheared in the
vertical and constant in the horizontal. The maximum inflow velocity is the same as in the
previous calculation, \(U = 30 \text{ cm s}^{-1} = H_0 M^2/\nu_0\), where \(H_0 = 500\) is the domain depth,
\(M^2 = (g/\rho_0) \nu_y = b_y\) is the horizontal stratification. The temperature change across the basin
is 0.6\(^\circ\) C at all depths, giving \(M^2 = 6 \times 10^{-8} \text{ s}^{-2}\). This calculation reproduces many of the key
features of the previous calculation. The average temperature between days 3 and 10 at 45 m
depth is shown in Fig. 29a. The temperature along the offshore boundary is uniform because
there is no heat loss there, while the temperature along the southern boundary decreases by
approximately 0.4\(^\circ\) C. As a result, the total change in density across the channel is less at the
outflow than at the inflow, implying a decrease in the vertical shear of the geostrophic velocity
at this depth. The average net vertical transport over the region \(x = 20 \text{ km}\) to \(x = 30 \text{ km}\) is
shown in Fig. 30 by the solid line. This profile looks very much like the downwelling in the
previous calculation. The net vertical motion is primarily downwelling in the upper ocean with
a maximum of \(1.11 \times 10^4 \text{ m}^3 \text{ s}^{-1}\) near 100 m depth, decreasing to zero at the surface and at the
The vertical section of the average zonal velocity is shown in Fig. 31a. The development of the no-slip boundary layers is evident along the northern and southern walls. The velocity in the interior, below the mixed layer, is nearly uniform at each depth, and close to the value at the inflow. There has been some increase in the interior velocities in response to the development of the no-slip boundary layers in order to conserve mass within the domain. Within the mixed layer, however, the velocity near the surface has decreased from its inflow value and the velocity near the base of the mixed layer has increased, particularly in the southern part of the domain. These adjustments make it difficult to distinguish between changes in the zonal velocity resulting from the buoyancy forcing and changes resulting from the development of the no-slip boundary layers. A calculation was carried out that had no surface forcing at all, but was otherwise identical to this calculation. The difference between the mean zonal velocity between $x = 20$ km and $x = 30$ km for these two calculations can be attributed solely to the buoyancy-forcing, and is shown in Fig. 31b. The depth of the mixed layer for the case with surface cooling is indicated by the bold white line. The zero contour is black. Most of the velocity change due to buoyancy-forcing is found within the mixed layer. The zonal velocity is decreased in the upper portion of the mixed layer and increased near the base of the mixed layer, resulting in weaker vertical shear throughout most of the mixed layer. Very close to the southern boundary, the zonal velocity is increased throughout the mixed layer. This is different from the behavior in the interior and is due to the development of a very narrow boundary layer, discussed next.

The non-hydrostatic layer

There is very intense downwelling right next to the southern boundary. The width of this
downwelling layer can be estimated following the approach of Stewartson (1957) (see also Pedlosky, 2009, and Chapter XX), who found that there are two narrow boundary layers required to transition a region of interior flow driven by stress at the surface and bottom to that of an adjacent flow driven at a different speed. A boundary layer of width $E^{1/4}$ exists to allow the geostrophic flow parallel to the boundary to smoothly transition from one regime to the other, where $E = A/f_0 L^2$ is the horizontal Ekman number, $A$ is the horizontal viscosity, and $L$ is a horizontal length scale. However, this transitional layer can not support the vertical mass transport that is required to match the upper and lower Ekman layers. This is achieved in a narrower, nonhydrostatic, boundary layer of nondimensional width $E^{1/3}$, or dimensional width $(AH/f_0)^{1/3}$, where $H$ is a vertical length scale. While the $E^{1/4}$ layer does not exist for the present problem, where the forcing is due to an along-boundary pressure gradient and not surface and bottom Ekman layers, the $E^{1/3}$ layer that carries the vertical mass transport does. The zonal pressure gradient, which was not considered in the original solution by Stewartson, does not alter the width of this boundary layer. For the values used here ($f_0 = 10^{-4}$ s$^{-1}$, $A = 1$ m$^2$ s$^{-1}$, $H = 100$ m) the horizontal scale of the downwelling region is predicted to be $O(100$ m).

The downwelling near the wall in the model is contained mostly within 1 grid cell of the boundary, so it is not well resolved with the standard 100 m grid. An identical calculation was carried out with the meridional resolution increased to 25 m between 0 and 200 m and 50 m between 200 m and 400 m from the boundary. The net vertical transport in this case is very similar to the standard resolution case, $1.24 \times 10^4$ m$^3$ s$^{-1}$ compared to $1.11 \times 10^4$ m$^3$ s$^{-1}$. The mean zonal, meridional, and vertical velocities between $x = 20$ km and $x = 30$ km near the southern boundary at 45 m depth are shown in Fig. 32. The downwelling is still concentrated within 100 m of the wall, so the horizontal scale of this downwelling region, while
only marginally resolved with the standard grid, is $O(100 \text{ m})$, consistent with that predicted by
the $E^{1/3}$ Stewartson layer. Linear interpolation of the vertical transport indicates that 80% of
the total downwelling occurs within 107 m of the boundary. A calculation with the horizontal
viscosity increased to $8 \text{ m}^2 \text{s}^{-1}$ results in 80% of the downwelling occurring within 204 m of
the boundary, in close agreement with the expected doubling of the boundary layer width for a
factor of 8 increase in viscosity.

A detailed analysis of this boundary layer in the linear limit by Pedlosky (2009) verifies that
the horizontal scale of the downwelling region, and the abrupt gradient in the along boundary
flow, scales as $E^{1/3}$. His analysis shows that it is the weak vertical stratification that is key to
the existence of this narrow, nonhydrostatic layer whose function is to bring the cross-channel
geostrophic flow to zero at the wall. This Stewartson layer is likely to be replaced by a bottom
Ekman layer if the vertical wall is replaced by a sloping bottom.

cooling distribution

This calculation with spatially variable cooling shows many similarities with the uniform
cooling case and spatially variable along-channel flow. The advantage of the spatially variable
cooling and uniform velocity is that other configurations can be employed that assist in our
understanding of what controls the net downwelling. A case with spatially uniform cooling
of 500 W m$^{-2}$ and uniform horizontal and vertical stratification results in density changes
along both the southern and northern boundaries (Fig. 29b). The magnitude of the change in
temperature along each boundary is similar, so that the net change in density across the channel
at the outflow is very similar to the net change in density at the inflow, even though the density
itself has increased. The net vertical mass transport between 20 km and 30 km is very small
(Fig. 30). Even though there is a mixed layer of $O(100 \text{ m})$ depth, and mixed layer instabilities
form and drive a restratifying cross-channel circulation, they do not drive a net vertical motion.
when integrated across the basin. A revealing calculation is obtained with a heat loss that is zero at the southern boundary and increases linearly to $1000 \text{ W m}^{-2}$ at the northern boundary. Now, the temperature is nearly constant along the southern boundary but decreases by approximately $0.4^\circ \text{C}$ along the northern boundary (Fig. 29c). The net vertical motion in this case is upward at about the same amplitude as the downwelling found in case with cooling enhanced along the southern boundary (Fig. 30).

A final calculation was carried out in which the heat loss increased from zero at the northern boundary to $1176 \text{ W m}^{-2}$ at 3 km from the southern boundary and was zero within 3 km of the southern boundary. The sea surface temperature is constant along the northern boundary, but is now also nearly constant along the southern boundary (Fig. 29d). Because there is no heat loss there, there is no means to support a strong pressure gradient and the along-channel velocity simply advects the isotherms downstream. The strong meridional cell adjacent to the southern boundary is not present in this case, again demonstrating its connection to the pressure gradient near the boundary. The net vertical motion is also very weak (Fig. 30). A similar sensitivity to surface insulation within 200 km of the boundary was found by Spall and Pickart (2001) for the basin-scale overturning circulation in a non-eddy resolving climate model. The present results suggest that this process will remain important for the basin-scale thermohaline circulation, even when the lack of convection is limited to within a few kilometers of the boundary. This result is also consistent with the modeling study of Walin et al. (2004), in which a baroclinic current was cooled and formed a barotropic boundary current yet resulted in no net downwelling. The form of their surface forcing resulted in no heat loss adjacent to the boundary, and was thus unable to support a pressure gradient, or downwelling, along the boundary.

The two calculations here that do not have a change in the density gradient across the channel both have the interior ageostrophic overturning cell driven by the mixed layer instabilities,
yet neither has any appreciable net vertical motion in the basin, demonstrating that these cells play no direct role in the net sinking in the basin. The strong cell near the southern boundary is also found in the case with uniform cooling, and there is no equivalent cell near the northern boundary (because the relative vorticity is positive there), yet there is no net vertical motion, demonstrating that this cell is not a significant component of the downwelling. Sinking is achieved when the density within the mixed layer increases along a boundary in the direction of Kelvin wave propagation, upwelling results when it decreases in the direction of Kelvin wave propagation.

*parameter dependencies*

The results in the previous section isolate the pressure gradient along the boundary as the key feature that controls net vertical motion. The pressure gradient on the boundary is related to the mixed layer depth and the density gradient along the boundary through the hydrostatic relation. Each of these calculations had the same net surface heat loss yet demonstrated completely different net vertical motions, clearly demonstrating that there is no direct relationship between heat loss and downwelling. The key to understanding the downwelling is to understand what controls the pressure gradient along the boundary. A simple model of the temperature within the mixed layer is now formulated to provide a framework with which to understand and predict how the buoyancy-forced downwelling will vary with environmental parameters.

For simplicity, it will be assumed that the pressure does not vary along the offshore side of the boundary current. For cyclonic boundary currents that encircle marginal seas subject to buoyancy forcing, this is roughly consistent with having the offshore edge of the boundary current being defined by an isotherm. The net downwelling is then determined by the lateral, large-scale flow into the very narrow nonhydrostatic layer adjacent to the boundary. An important assumption here is that the boundary layer exists in order to satisfy the no-normal flow
boundary condition and conserve mass, and that the pressure gradient is set by the flow in the boundary current just outside the narrow boundary layer. It is also assumed that all of the transport towards the boundary layer downwells within the boundary layer. This is in close agreement with the numerical results, and is also supported by the linear theory of Pedlosky (2009).

Consider the buoyancy balance near the southern boundary within the mixed layer but outside the nonhydrostatic layer of width $E^{1/3}$. If the along-channel velocity at the base of the mixed layer is $U$, the mixed layer depth is $h$, the mixed layer buoyancy is $b = -g\rho/\rho_0$, and the surface buoyancy flux $B = \alpha g Q/\rho_0^2 C_p$, then the density equation within the mixed layer may be written as

$$Ub_x = -\frac{B}{h},$$

(42)

where $\alpha$ is the thermal expansion coefficient, $g$ is the gravitational acceleration, and $C_p$ is the specific heat of seawater and the variables $U$, $b$, and $h$ are functions of downstream distance $x$ only. This is a balance between the along-boundary advection of buoyancy and surface cooling. If it is assumed that the mixed layer is an unstratified layer overlaying a uniformly stratified region below, the depth of the mixed layer can then be related to the buoyancy as

$$h = -\frac{b}{N^2},$$

(43)

where $N^2 = b_z$ is the Brunt-Väisälä frequency and $b$ is taken to be relative to the surface buoyancy in the absence of any cooling ($h = 0$ when $b = 0$). Combining with (42), the buoyancy gradient in the along channel direction can be written as

$$b_x = \left(\frac{BN^2}{2Ux}\right)^{1/2},$$

(44)
The downstream buoyancy gradient increases with increasing cooling, as expected. However, the buoyancy gradient also depends on the along-channel velocity because the balance is between horizontal advection of buoyancy and surface cooling. The dependence on stratification enters because the mixed layer will be shallower for stronger underlying stratification, and the buoyancy change will be larger for a shallower mixed layer.

Spall and Pickart (2001) considered the geostrophic flow within a mixed layer subject to cooling and found that, for a density that is increasing downstream, the flow will be towards the boundary in the upper half of the mixed layer and away from the boundary in the lower half of the mixed layer. Thermal wind gives a maximum downwelling at the mid-depth of the mixed layer, per unit along boundary distance, of

$$W = -\frac{b_x h^2}{8f_0}. \tag{45}$$

This expression was found to compare well with a series of low resolution, basin-scale wind and buoyancy-driven general circulation models.

If the mixed layer depth were known, the downwelling rate could be derived from (42) and (45) to be

$$W = \frac{B h}{8f_0 U}. \tag{46}$$

It is clear from (43) that the mixed layer depth will increase downstream as the boundary current is cooled and the buoyancy decreases. Equation (45) may be combined with (43) and (44) to provide an estimate of the downwelling that varies with downstream position as

$$W = \frac{1}{4f_0} \left( \frac{B^3 x}{2N^2 U^3} \right)^{1/2}. \tag{47}$$
Although this expression is more complicated than (46), it is also more revealing regarding the competing effects that influence net downwelling. The downwelling increases with increasing cooling, as expected, but it also increases with increasing distance, decreasing velocity, decreasing stratification, and decreasing Coriolis parameter. Downwelling depends on distance because the mixed layer depth increases with distance downstream. Less downwelling is found for stronger boundary currents because the pressure gradient is less due to stronger horizontal advection balancing the surface cooling, but it is also due to the fact that a stronger horizontal advection limits the depth of mixing, (42). The downwelling also increases with decreasing stratification because the mixed layer will penetrate further for the same cooling rate. The increasing downwelling with decreasing Coriolis parameter is simply due to the geostrophic balance resulting in more flow toward the boundary to balance a given pressure gradient.

A series of model calculations was carried out by Spall (2008) in order to test the parameter dependencies predicted by (47). The model was forced with uniform horizontal and vertical stratification and a heat loss that was maximum at the southern boundary and linearly decreased to zero at the northern boundary. The magnitude of the surface cooling, vertical stratification, along-channel geostrophic flow, and Coriolis parameter were each varied (see Spall, 2008 for details). Each of these model calculations was carried out with 200 m horizontal resolution and 10 m vertical resolution, however the circulation features are very similar to the previously discussed calculation with higher resolution. The maximum net downwelling per unit along boundary distance is also similar (1.11 m$^2$ s$^{-1}$ for the high resolution calculation and 1.04 m$^2$ s$^{-1}$ for the low resolution calculation). The maximum downwelling rate was calculated, as in Fig. 30, for each of these calculations and is compared with the theory in Fig. 33. The velocity scale used in (47) has been taken to be proportional to the surface geostrophic velocity at the inflow, $U = cH_0M^2/f_0$, where the constant $c = 0.43$ produces a least square
fit line to the data with slope 1. It is expected that $c < 1$ because the velocity decreases within the no-slip boundary layer, which is much wider than the downwelling layer. Nonetheless, the geostrophic scaling allows for a systematic means to estimate the influence of the horizontal velocity on the downwelling and makes clear the dependence on the controlling parameters $M^2$ and $f_0$.

The agreement between the downwelling diagnosed from the model and that predicted by the simple theory is quite good. The central calculation is indicated by the square (low resolution) and star (high resolution), the two are nearly indistinguishable on this scale. It is interesting that all but three of these calculations are subject to the same heat loss at the surface yet the net downwelling varies by a factor of 6.

It is somewhat counterintuitive that the simple theory (47) indicates that the total downwelling does not depend on the offshore extent of the boundary current or the amplitude or pattern of cooling away from this near boundary region. To demonstrate this independence, the model was run with a maximum heat loss of $1000 \text{ W m}^{-2}$ at the southern boundary that linearly decreased to zero at the northern boundary in a channel 40 km wide, twice as wide as in the standard case. The horizontal velocity, horizontal stratification, and vertical stratification were the same as the standard case, but due to the wider domain the total heat loss was twice as large. The total net downwelling in this case was $1.08 \times 10^4 \text{ m}^3 \text{ s}^{-1}$, essentially the same as for the 20 km wide channel. The downwelling is independent of the current width provided that the current transports enough heat to balance the surface cooling and maintain the along-boundary baroclinic pressure gradient.

It does not appear to be necessary to resolve the nonhydrostatic physics and convective plumes explicitly. A hydrostatic calculation with horizontal viscosity and diffusivity increased to $5 \text{ m}^2 \text{ s}^{-1}$, and with vertical convection parameterized by increasing the vertical diffusivity
to $1000 \text{ m}^2 \text{s}^{-1}$ for unstable profiles, results in a net downwelling of $1.10 \times 10^4 \text{ m}^3 \text{s}^{-1}$, close to the standard calculation. The subgridscale mixing is sufficiently large that the symmetric instabilities are suppressed, but all other aspects of the zonally averaged flow are similar to the non-hydrostatic result. The density within the mixed layer is essentially uniform with depth. The mean cross-channel flow in the interior is now towards the boundary in the upper mixed layer and away from the boundary in the lower mixed layer, as expected from geostrophy (Spall and Pickart, 2001). The ageostrophic cell near the southern boundary is also found, supporting the interpretation that this is not resulting from an instability of the mixed layer.

The underlying circulation that redistributes mass in the vertical is more clearly revealed by considering a hydrostatic calculation with free slip boundary conditions and increased viscosity and diffusivity of $5 \text{ m}^2 \text{s}^{-1}$. This suppresses the strong nonlinear recirculation gyre near the boundary in addition to the symmetric instabilities in the interior. The resulting net downwelling rate is $0.75 \times 10^4 \text{ m}^3 \text{s}^{-1}$, very close to that found with free slip boundary conditions, nonhydrostatic physics, and low viscosity and diffusivity. The along channel velocity is a maximum adjacent to the southern boundary because the no-slip boundary layer is no longer present (Fig. 34a). However, the cross-channel velocity is now dominated by flow towards the southern boundary over the upper mixed layer and flow away from the boundary in the deep mixed layer and just below the mixed layer. Note that the magnitude of this cross-channel flow is less than that found when symmetric instabilities are present, but it is just what is required to provide the net downwelling rate of $O(1 \text{ m}^2 \text{s}^{-1})$ along the boundary. The magnitude of the meridional velocity increases towards the boundary in both the upper and lower mixed layer. The vertical velocity, as implied by the streamfunction in Fig. 34d, is near zero over most of the interior of the basin. This indicates that the change in $v$ towards the southern boundary is gained largely at the expense of the along-channel velocity, not by upwelling and/or downwelling in the interior.
The downwelling is localized primarily within 1 grid cell of the southern boundary, although there is weaker downwelling within approximately 1 km of the southern boundary. This profile is very similar to that predicted by the linear theory of Pedlosky (2009). This calculation demonstrates that the redistribution of mass in the vertical is achieved by a geostrophic flow towards the boundary in the upper mixed layer, downwelling very close to the boundary, and a return flow away from the boundary below the mixed layer. Thus, while the acceleration at the base of the mixed layer is physically very close to the deceleration at the surface, the water parcels had to make a relatively long traverse all the way to the narrow boundary layer in order to sink to the deeper depth.

4. Summary

Buoyancy-forced marginal seas are the source regions for many of the dominant water masses in the World Ocean. While relatively small in geographic extent, these basins are the locations of most regions of deep convection and produce water masses whose signature can be seen far from their source. The properties of the product waters produced in such marginal seas are of much interest for the general oceanic circulation, understanding its sensitivities to changes in atmospheric forcing and climate, and for our ability to properly represent these processes in large-scale climate models. In Section 2, simple dynamic and thermodynamic constraints were combined to provide analytic estimates of the key properties of the exchange between semi-enclosed marginal seas and the open ocean. The key dynamical components are a mean boundary current that exchanges mass and tracers between the marginal sea and the open ocean and baroclinic instability that controls the exchange of buoyancy between the boundary current and the interior of the marginal sea. It was shown that the boundary current arises be-
cause of the buoyancy forcing in the marginal sea, and its strength is determined by the eddy exchange dynamics. These two conditions, along with mass conservation, buoyancy budgets, geostrophic momentum balances, and an eddy flux parameterization, allow for direct estimates of the density of the convective waters, density of the exported waters, exchange rate between the marginal sea and the open ocean, net downwelling in the marginal sea, and in some cases the heat flux into the marginal sea. A key aspect of these solutions is that the mean flow through the interior of the marginal sea is negligible. This is no longer true if topographic contours directly connect the interior of the marginal sea with the open ocean, in which case eddy fluxes are much less important (e.g. Spall, 2005).

The dynamics that support net vertical motions that result from buoyancy loss at the surface were discussed in Section 3. Three distinct parameter regimes can be identified: dissipative, stratified; weakly dissipative, stratified; unstratified. While the detailed dynamical balances in each of these regimes vary, there are some common elements. The net vertical motion resulting from buoyancy loss in the basin is downward and is located near the lateral boundaries. The magnitude of the downward mass transport is just what is required to maintain a thermal wind balance in the along-boundary geostrophic flow. Because buoyancy has been extracted from the basin, the horizontal density gradient is reduced and the transport in the boundary current is shifted from the baroclinic mode to the barotropic mode as one moves cyclonically around the marginal sea. This vertical motion introduces stretching into the vertical vorticity equation, which is ultimately balances by lateral dissipation into the side boundary. The details of these viscous lateral boundary layers vary, depending on stratification and resolved physics. In all cases, the region of net downward motion is distinct from the region where buoyancy is extracted from the fluid and where there is a net diapycnal mass transport from lighter to denser density surfaces.
The dynamics of downwelling motions present some challenges for both the observational and large-scale climate modeling communities. The vertical velocities are highly variable in space and time, making direct observations of net vertical mass transport difficult. Perhaps the best approach would be to measure inflowing and outflowing transports along a segment of a boundary current that is subject to buoyancy loss. An initial attempt in this direction was carried out by Pickart and Spall (2007). However, synoptic variability is likely to be large relative to the net transports, so long time and high resolution measurements are required. Global-scale climate models will not be able to directly resolve the processes that control buoyancy-forced downwelling for some time. However, there is hope in that the net downwelling is determined by the deformation-scale flow, and the details of the downwelling dynamics do not need to be resolved. So, progress will rely on getting the mean boundary currents correct (velocity, mixed layer depth, along-boundary variations in density). Since much of the heat balance within the boundary currents involves eddy fluxes with the interior, this will require an accurate eddy flux parameterization. The eddy-resolving calculations presented here suggest that parameterizations should: be adiabatic; identify regions of enhanced eddy fluxes (including the role of topography); and transport tracers far from the boundary current. This last constraint will be difficult to achieve since it requires a non-local eddy flux parameterization and knowledge of how eddies lose their heat to the atmosphere and surrounding ocean.

Acknowledgments.

The work summarized in this paper was supported by the National Science Foundation under Grants OCE-0240978, OCE-0726339 and OCE-0850416, and by the Office of Naval Research under Grants N00014-01-1-0165 and N00014-03-1-0338. Joe Pedlosky, Fiamma Straneo, and Bob Pickart are acknowledged for numerous fruitful discussions on a wide range of
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