Timescales for stratospheric transport inferred from tracers

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How to diagnose large-scale transport? Long-lived tracers.

Important class: sources in troposphere lead to time-varying tropospheric mixing ratio. Signal enters stratosphere (overworld) in tropics. In stratosphere, tracers ~ inert.

Observation platforms: satellite, balloon, aircraft

Examples: $\text{CO}_2$, $\text{SF}_6$, $\text{H}_2\text{O}$

Goal is to infer tracer-independent transport timescales from tracers.
Example: steadily increasing tracers, $\text{CO}_2$, $\text{SF}_6$.

Stratospheric concentration lags troposphere.

Call lag-time “age”.

Tempted to interpret as transit-time of air parcel from troposphere.

$$q(r,t) = q_{trop}(t \square \mathcal{G}(r))$$
Periodic tracers: e.g., H$_2$O imprint of seasonal variations of tropical tropopause temperature. Propagates upward.

HALOE H$_2$O (Mote et al, 1996; 1998)

Can define $\max(r,t) = \max_{trop} (t \square \square (r))$

Is $\square$ a transit-time of an air parcel?
Lag-time of linear tracer much larger than phase lag of annually-periodic tracer.

(Originally in literature would find statements that both tracers should be direct measure of tropical upwelling.)
“Age spectrum” as interpretive framework:  

\[ \frac{\partial \Box}{\partial t} + L(\Box) = 0 \quad \text{with inhomogeneous} \quad \Box(\Box, t) = f(t) \]

for linear transport operator \( L \) (e.g., advection-diffusion)

\[ \frac{\partial G}{\partial t} + L(G) = 0 \quad \text{with} \quad G(r, t | \Box, t') = \Box(t | \Box, t') \]

Linearity: \( \Box(\Box, t) \) expressed as linear combination of \( \Box(t) \) then \( \Box(r, t) \) expressed as same linear combination of \( G(r, t | \Box, t') \).

\[ \Box(r, t) = \int_0^t dt' \Box(\Box, t') G(r, t | \Box, t') \]

If transport in steady-state

\[ \Box(r, t) = \int_0^t dt' \Box(\Box, t \Box t') G(r, \Box, t') \]
Age spectrum physical interpretation: PDF of transit-times

\[ \int_0^\infty dt \, G(r, t) = 1 \]

Mass fraction of parcel at \( r \) that made last troposphere contact \( t \) to \( t + \Delta t \) ago.

Narrowly-peaked early times near tropical tropopause.
Broad, older, asymmetric far from tropical tropopause.

Variance of age spectrum is measure of mixing.
E.g., 1-D advection-diffusion:

\[ \Box^2(x) = \frac{kx}{u^3} \]
Easy to simulate $G$ in a GCM. Apply $\bar{C}(t)$-like BC on concentration at surface (stratospheric response not sensitive to details). Run 10-20 years. Time-series Response (normalized in time) at each stratospheric $r$ is $G(r,t)$.

NCAR MACCM2 (Hall et al., 1999)

Characteristic shape: early peak, long tail; well-fit by two-parameter form (next lecture).
Linear tracer \( q_{\text{trop}}(t) = t \)

\[
q(r,t) = \int_0^t (t - t')G(r,t')dt' = t \square(r)
\]

where \( \square(r) = \int_0^t dt G(r,t) \)

Lag-time of linear tracer is first-moment of \( G \), the “mean age”, independent of tracer growth rate.

How linear to get mean age?

Exponential tracer \( q_{\text{trop}}(t) = e^{rt} \)

define \( q(r,t) = q_{\text{trop}}(t \square) \)

Find \( \square(r) \square(r) \square^2(r) \)

where \( \square^2(r) = \int_0^t dt (t \square(r))^2 G(r,t) \)

if \( \square^1 >> \square^2/\square \) then \( \square = \square \)

In GCM studies, \( \square^2/\square \sim 0.5 - 1.5 \) year. Anthropogenic tracers grow more slowly than that, yield mean age.
$G(r,t)$

Past times

$\text{SF}_6$ and $\text{CO}_2$ (annual cycle removed) growth timescale longer than typical "width" of $G$ (as observed in GCMs).

So, 

$$[SF_6](r,t) = \left[SF_6\right]_{trop}(t \square \square)$$

Note: "trop" somewhat vague. Ideally, tropical tropopause. In practice, often some surface region. Makes 0.5-1.0 year difference.
Observations of “mean age”

Bulges up more than N₂O, CH₄
Segue ...

CCl$_y$ → Cl$_y$ and CBr$_y$ → Br$_y$ with time spent in active photolysis regions. Expect some correlation with mean age. (But imperfect, because age doesn’t provide pathway information).

Romashkin et al., 2000; SOLVE data

Total Cl and total Br are inert. Time-varying signals propagate from troposphere, so correlation with mean age limited only by weak non-linearity of time-variation. Signals has leveled-off late 1990s, so age-dependence weak.
$\text{Cl}_{\text{tot}}$ variation nonlinear enough to be sensitive to shape of age spectrum

Use 2-parameter “inverse Gaussian” form

$$G(t | \mu, \sigma) = \frac{1}{2 \sigma \sqrt{\pi} \tilde{t}^3} \exp \left[ - \frac{2(\tilde{t} - 1)^2}{4 \tilde{t}^2} \right]$$

Better fit to observed for $\mu = \tilde{t}$ than simple lag by $\mu$.

(But still don’t understand abruptness of turnover.)
Periodic tracer of period $T$. 

$$q_{\text{trop}}(t) = \exp(i2\pi t/T)$$  
$$q(r,t) = A(r)\exp(i2\pi(t - \tau(r))/T)$$

if $T >> 2\pi^2/\tau \approx 6 \text{ years}$ then $\tau = \tau$

Lag-time of annually-periodic tracer ($\text{H}_2\text{O}, \text{CO}_2$) are not mean age.

To interpret periodic (and other) tracers develop and apply simple model …
“Tropical leaky pipe” model
(Plumb, 1996; Neu and Plumb, 1999)


\[
\frac{\partial q_T}{\partial t} + W \frac{\partial q_T}{\partial Z} e^{Z/H} \frac{\partial}{\partial Z} K_T e^{Z/H} \frac{\partial q_T}{\partial Z} = \nabla\left(q_T \nabla q_T\right)
\]

\[
\frac{\partial q_M}{\partial t} + W \frac{\partial q_M}{\partial Z} e^{Z/H} \frac{\partial}{\partial Z} K_M e^{Z/H} \frac{\partial q_M}{\partial Z} = (\nabla + \nabla)(q_T \nabla q_T)
\]

where \(\nabla = \nabla e^{Z/H} \frac{\partial}{\partial Z} \left(e^{Z/H} W(Z)\right)\) is tropical divergence rate

and \(\square = \frac{M_T}{2M_M}\) is measure of tropical barrier latitude.
Isolated tropics:
Plug-flow up the tropical pipe.

\[
\square = 0 \quad K_T = 0 \quad \square = \square = t_{adv}
\]

Allow recirculation:
Recirculated air averages several cycles of annually-periodic tracer, but contributions to mean age add.

\[
\square = 1\text{yr}^{-1} \quad K_T = 0 \quad \square > \square \quad t_{adv}
\]

Recirculation and weak vertical mixing:
No significant change, except that Age spectra look more “realistic.”

\[
\square = 0 \quad K_T = 0.1\text{m}^2\text{s}^{-1} \quad \square = \square \quad t_{adv}
\]

For small \( K_T \):

\[
A(Z) = \exp \left[ \frac{\int_0^Z W(Z')dZ'}{W(Z)} \right]
\]

\[
\square (Z) \quad t_{adv} = \frac{\int Z dZ'}{W(Z')}
\]
Mote et al. (1998) constrain 3-parameter version of TLP (W, K, \(s\)) with fixed midlatitude values using H\(_2\)O amplitude and phase and CH\(_4\) observations (with radiative-chemical model). Obtain best-fit parameter values as function of Z.

Solutions are \(~\) in weak K limit. Vertical diffusion plays secondary role in transporting, attenuating signal.
What do Mote’s solutions imply for tropical-midlat exchange? Use best-fit $W(Z)$, $\tilde{W}(Z)$ in full tropical-midlatitude version of TLP.

High $dW/dZ$ means minimum in tropical mass divergence (and no large mixing to counter). Fewer “up-and-over” paths cross tropical barrier at this height. Associated transit-time with minimum probability.
Empirical midlatitude age spectra: bimodal spectra best fit CO$_2$ data.
Age spectrum does not provide path information.
What is relationship between path height and transit-time?

Define “maximum-path-height” distribution $Z(r,t|z)$ such that
$Z(r,t|z)\,dz = \text{mass fraction of parcel at } (r,t) \text{ that reached maximum height } z \text{ to } z+dz \text{ since last tropopause contact.}$

And, joint “maximum-path-height-transit-time” distribution $P(r,t|z,x)\,dz\,dx$ such that
$P(r,t|z,x)\,dz\,dx = \text{mass fraction of parcel at } (r,t) \text{ that was last at tropopause time } x \text{ to } x+dx \text{ ago and reached maximum height } z \text{ to } z+dz.$

Marginals:
$\int_0^\infty dt P = Z$
$\int_0^\infty dz P = G$

Normalized:
$\int_0^\infty dz \int_0^\infty dt P = 1$
Joint distributions in lower midlatitude TLP

Minimum in MPH distribution. No tropical divergence at this height, so unlikely to have trajectory with this maximum height.

Upper and lower components of MPH distribution illustrate partitioning of trajectories to midlatitudes into two classes: “direct” and “up-and-over”. About 50% of air in each class.
Other topics

• Tracers with stratospheric source, tropospheric sink (e.g., radiocarbon) and relationship to aircraft emissions.

• Tracers to infer polar vortex descent, degree of isolation.

• Interannual variability as seen in tracers (QBO, internal variability, GHG-forced trends).

• Timescales for ultimate mixing at small-scales.
More general case of boundary propagator:
spatial variation of boundary condition and non-steady flow.

\[
\mathcal{P}(r, t) = \int_{t'}^{t} dt' \int d^2 r' \mathcal{P}(r', t') G(r, t | r', t')
\]

Still have probabilistic interpretation:

\[G(r, t|r', t') d^2 r' = \text{probability that parcel at } (r, t) \text{ made last contact time } t' \text{ to } t'+dt' \text{ and made the contact on } d^2 r'.\]

\[G(r, t|r', t') = \text{joint PDF in source time and space.}\]

Note: much more difficult to compute, now.
Need \(d(t)\) BC for each \(t'\) and \(r'\) to resolve on source.
Example: surface origins and transit-times for tropospheric air
Ocean illustration: simulate \( G(r, r', t-t') \) in North Atlantic MYCOM.  
(Haine and Hall, 2002)

Tile the domain. For \( i^{th} \) tile, tracer has BC = \( \mathbb{H}(t) \) and zero elsewhere.